Analysis of the Naval Observatory Flagstaff Station 1-m telescope using annular Zernike polynomials

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Abstract. The Naval Observatory Flagstaff Station 1-m telescope is evaluated for the addition of adaptive optics capabilities to its instrumentation suite. Zernike decomposition of the optical system based on phase diversity measurements shows that the static optical aberrations are small enough that they will not degrade the performance of the deformable optical element. The analysis makes use of annular pupil Zernike polynomial reconstruction of the wavefront to accommodate the large obscuration in this telescope and compares this with the results from using filled circular Zernike polynomials. [DOI: 10.1117/1.1598210]

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1 Introduction

The Naval Observatory Flagstaff Station (NOFS) 1-m telescope was the last telescope to be designed and built by G. W. Ritchey and is a Ritchey-Cretien Cassegrain optical design. The telescope was completed in 1934 and was originally located in Washington, D.C. The telescope was moved to Flagstaff, Arizona, in 1955 to take advantage of the darker skies. This telescope has been the mainstay of work at the Flagstaff Station and, in keeping with this role, it is now being evaluated for upgrading to high angular resolution work by adding an adaptive optics system.

Other telescopes built by Ritchey include the 60- and 100-in. telescopes at the Mount Wilson Observatory, which have both had adaptive optics systems retrofitted to them at various times in the past. These telescopes are conventional Cassegrain telescopes with obscurations less than about 28%. The unique feature of the NOFS 1-m telescope that makes the analysis more interesting is that the secondary mirror is particularly large, having an approximately 45% obscuration, which was designed to provide an unvignetted field of view of greater than a degree. While it has been known for many years that the use of filled circular Zernike polynomials on telescopes with large obscurations can lead to significant errors, a recent literature search has shown that few papers exist on the use of the more appropriate annular Zernike polynomials[1] for analysis.

This paper presents an analysis of the existing optical system of the NOFS 1-m telescope to evaluate whether the static aberrations in the existing telescope optics would reduce the effectiveness of an adaptive optics system. These measurements are based on wavefront measurements using the technique of phase diversity[2] with Zernike polynomial decomposition used to evaluate the telescope's wavefront quality. An annular Zernike polynomials expansion was chosen for this analysis because of the telescope's large obscuration and the results are compared with using conventional filled circular Zernike polynomials.

2 Telescope Wavefront Analysis

The ''as built'' prescription for the NOFS 1-m telescope was evaluated in Zemax and was used to simulate the wavefront characteristics of the telescope. This analysis showed that the peak-to-valley wavefront error was better than $\lambda/5$. In these calculations, the distance from the secondary mirror to the focal plane was optimized for the best focused spot size for this simulation. It was also clear from the Zemax® model that the telescope's large obscuration represented a significant deviation from the assumption of ''no obscuration'' for the use of filled circular Zernike polynomials. To analyze the wavefront, the technique of phase diversity was selected and the results from filled circular and annular pupil Zernike polynomial decomposition are compared in the evaluation of this wavefront.

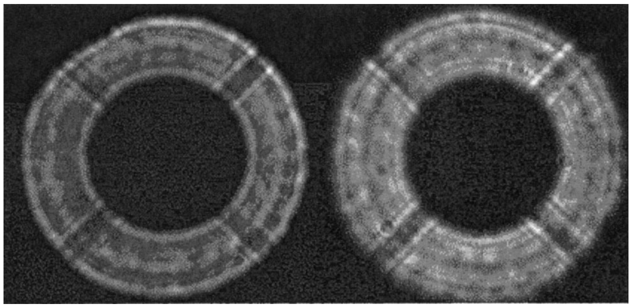

Fig. 1 Raw out-of-focus images taken at the NOFS 1-m telescope.

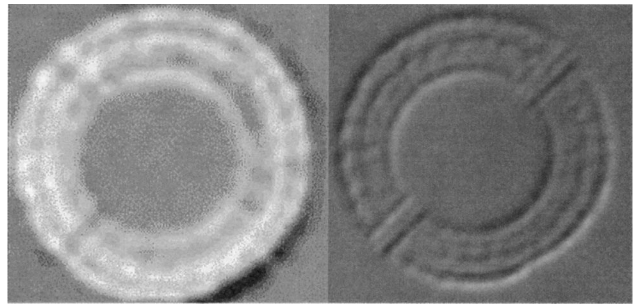

Fig. 2 Left: difference of the raw out-of-focus images of Fig. 1, an approximation of the $\partial I/\partial z$ component of Eqs. (1) and (2); Right: phase map calculated from the left image using Eq. (2). Notice that there is an enhancement of the spiders in one direction while they are lost in the orthogonal direction.

Stellar images were obtained from the NOFS 1-m telescope, recorded both inside and outside focus, as shown in Fig. 1, for evaluation using the phase diversity technique.[2] The objective was to analyze the wavefront that is delivered from the telescope to determine whether or not there were significant optical aberrations present in the telescope. For this analysis, we considered the transport equation for intensity and phase,[3,4] as shown in Eq. (1).

$$-k\frac{\partial I}{\partial z}=\nabla\cdot(I\nabla\phi)=\nabla I\nabla\phi+I\nabla^2\phi, \tag{1}$$

where I is the intensity, φ is the phase, $k=2\pi/\lambda$, and z is the propagation axis. The derivative along the optical axis is approximated with a finite difference. In the most common case, the two intensity maps recorded from both sides of the Gaussian focus are used to generate $\Delta I/\Delta z$. This is used to solve Eq. (1). Assuming a nearly constant intensity map in the plane of observation reduces Eq. (1) to

$$-k\frac{\partial I}{\partial z}=I\nabla^2\phi, \tag{2}$$

where Eq. (2) is the usual form of the curvature sensing equation.[2,4]

The images in Fig. 1 were obtained at the telescope by recording the out-of-focus images from a bright star. These images were taken at comparable distances from the best focus position. The images were recorded using an exposure time considerably longer than the "coherence time of the atmosphere" to average out the contributions of the atmospheric turbulence. Typical values for the "coherence time of the atmosphere" at a good site range around 10 m.

Figure 2 shows the difference of the two intensity images, corresponding to $\Delta I/\Delta z$ on the left and the phase image calculated from it using Eq. (2) on the right. In Fig. 2 (right) only two spiders can be seen after the conversion from an intensity image as compared to the four that are easily seen in the raw images in Fig. 1. This can be attributed to the presence of astigmatism enhancing the secondary spider support's footprint along one diagonal and reducing the phase contrast of the opposite diagonal.

It is convenient to decompose the phase wavefront into an orthonormal basis set (Zernike modes) to evaluate the wavefront. For sake of simplicity in plotting the decomposition of the wavefronts we count the Zernike modes in a sequential manner instead of using the customary 2-D indexing; the indexing order is shown in Fig. 3, beginning in the lower left corner of the plot, where n and $|m|$ are the radial and azimuthal indices, respectively.

Traditionally, filled circular Zernike polynomials are used for this type of decomposition and analysis. In the case of the NOFS 1-m telescope, the large obscuration would suggest that annular Zernike polynomials would be more appropriate, since these polynomials are an orthonormal basis set for an annulus. To examine the difference between these two variants of Zernike analysis the measured wavefronts are decomposed into the amplitudes of the coefficients of the modes, which are plotted as bar charts in Figs. 4 and 5. These values were calculated using custom software built in IDL (Ref. 3) and the mode numbering used is that shown in Fig. 3.

The measured wavefronts are composed of optical aberrations from the telescope misalignments and static aberrations in the optics themselves, while atmospherically induced aberrations are minimized by the averaging effects of the exposure time. The aberrations induced by alignment errors cannot be easily separated from the static optical aberrations in this case and are assumed to be much smaller than the atmospherically induced ones. Comparing these two figures it can be seen that the filled circular case shows

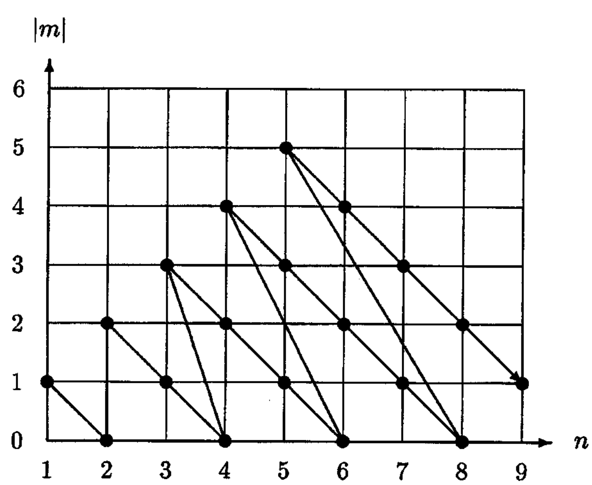

Fig. 3 Single index ordering of the 2-D Zernike array used compared to the customary 2-D indexing. The sequence begins in the lower left of the plot.

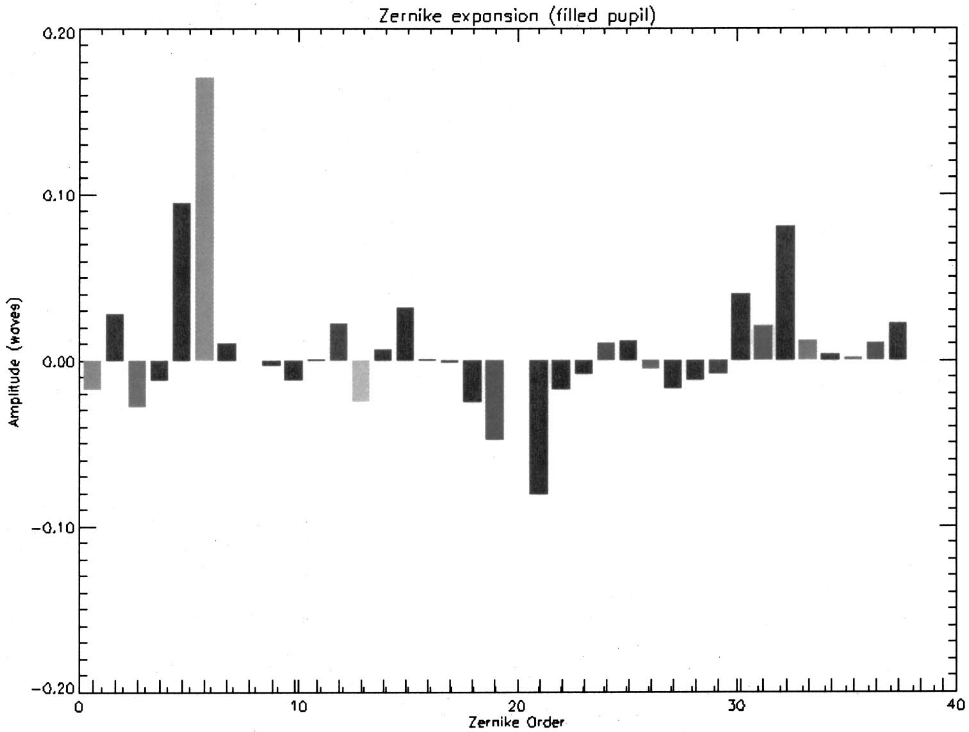

Fig. 4 Magnitude plot of the coefficients for the filled circular Zernike polynomial modes.

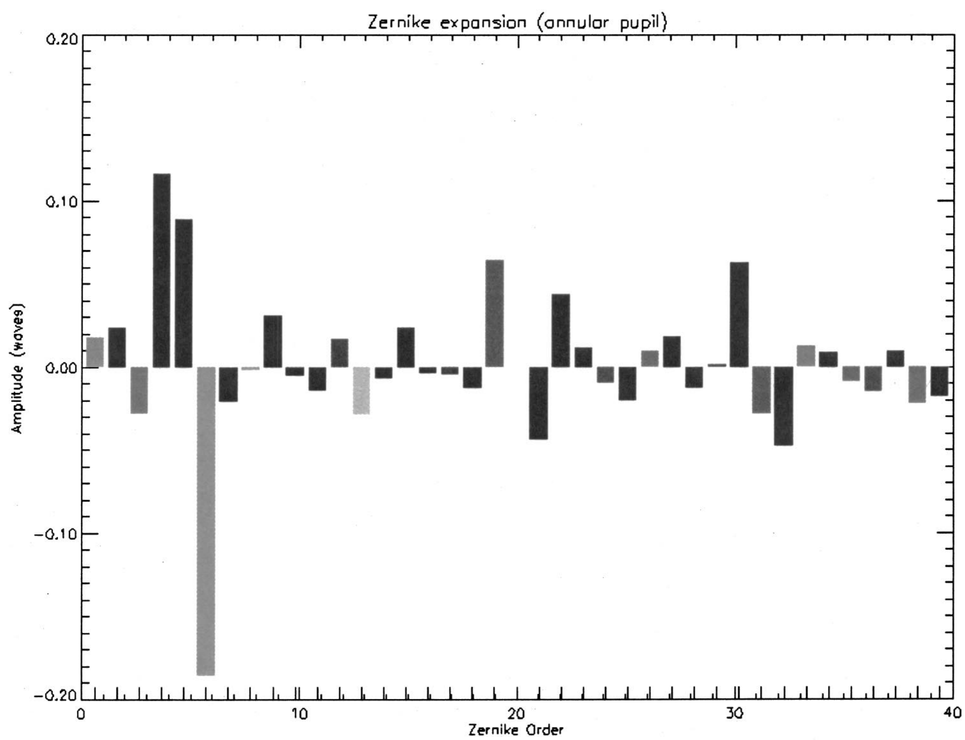

Fig. 5 Magnitude plot of the coefficients for the annular pupil Zernike polynomial modes.

Table 1 Comparison of filled circular versus annular pupil Zernike polynomials.

Spherical aberration	$A_0^4(6\rho^4-6\rho^2+1)$	$\frac{A_0^{4'}}{\sqrt{2}(1-\varepsilon^2)^2}[6\rho^4-6(1+\varepsilon^2)\rho^2+1+4\varepsilon^2+\varepsilon^4]$
Coma	$A_1^3(3\rho^3-2\rho)\cos\theta$	$A_1^{3'}\left(\rho^3-\frac{2}{3}\frac{1+\varepsilon^2+\varepsilon^4}{1+\varepsilon^2}\rho\right)\cos\theta$
Astigmatism	$A_2^2\rho^2\cos 2\theta$	$A_2^{2'}\frac{(2\cos 2\theta-1)\rho^2}{(1+\varepsilon^2+\varepsilon^4)^{1/2}}$
Defocus	$A_0^2(2\rho^2-1)$	$A_0^{2'}[\rho^2-(1+\varepsilon^2)/2]$

several different effects. The relative importance of the modes between the two Zernike expansions is similar, that is, the fractional weight of the amplitude of the modes are comparable, however, it is significant that in many of the modes, the associated sign has reversed. As examples, in the first five terms, modes 1 and 4 show opposite signs when the two plots are compared. The effect of using filled circular versus annular pupil Zernike modes is discussed in the next section.

3 Discussion

The large obscuration of the NOFS 1-m telescope was the original motivation for examining the use of annular pupil Zernike polynomials[1] in the analysis. Table 1 compares the form of selected filled circular Zernike polynomials with the annular pupil Zernike polynomials. In the table, $A_{|m|}^n$ is the coefficient of the mode, ρ is the radial component, θ is the angular component, and ε is the fractional diameter of the obscuration. In Table 1 we identify the Zernike mode with the correspondent balanced aberration as is customarily done (see, for example, Ref. 5).

The effect of the use of the annular pupil Zernike representation is compared with the filled circular Zernike for the two cases of spherical aberration and astigmatism shown in Fig. 6. In these two cases, the change in the magnitude of the polynomial can be clearly seen, however, the astigmatism plot shows how the sign change for the mode occurs. In adaptive optics systems, this is not a particularly important effect as typically the wavefront analysis does not use the Zernike modes explicitly but rather sets the deformable element to null out the wavefront error. However, this would be important in the case of determining the prescription of the optic.

Figure 7 shows the wavefront that is formed by recombining 40 annular pupil Zernike modes from the phase diversity data following Eq. (2). In this image, several low-order features can be seen including evidence of a mixture of trifold coma and astigmatism. This aberration is also evident in the amplitudes of these modes shown in Fig. 5. The root mean square (rms) value of the difference between the phase map shown in Fig. 2 (right) and the two Zernike expansions, full pupil and annular pupil is $\mathrm{rms}_{\mathrm{ann}}=0.05$ and $\mathrm{rms}_{\mathrm{full}}=0.08$, respectively, both with a standard deviation of 0.002. In either of the Zernike models, the rms wavefront errors are quite small, which enables their correction using deformable mirrors or spatial light modulators to correct these terms easily. However, should there be a desire to retouch the optical components themselves, how the coefficients are distributed across the two reflective elements of the telescope should be determined.

Based on this analysis, the amount of static aberration[5] present in this telescope is roughly $\lambda/4$ in magnitude and mostly concentrated in the low-order aberrations, as seen in Fig. 5. For most adaptive optics systems, static errors of this order can be easily corrected without compromising the dynamic range of the system. Much of the concern for this

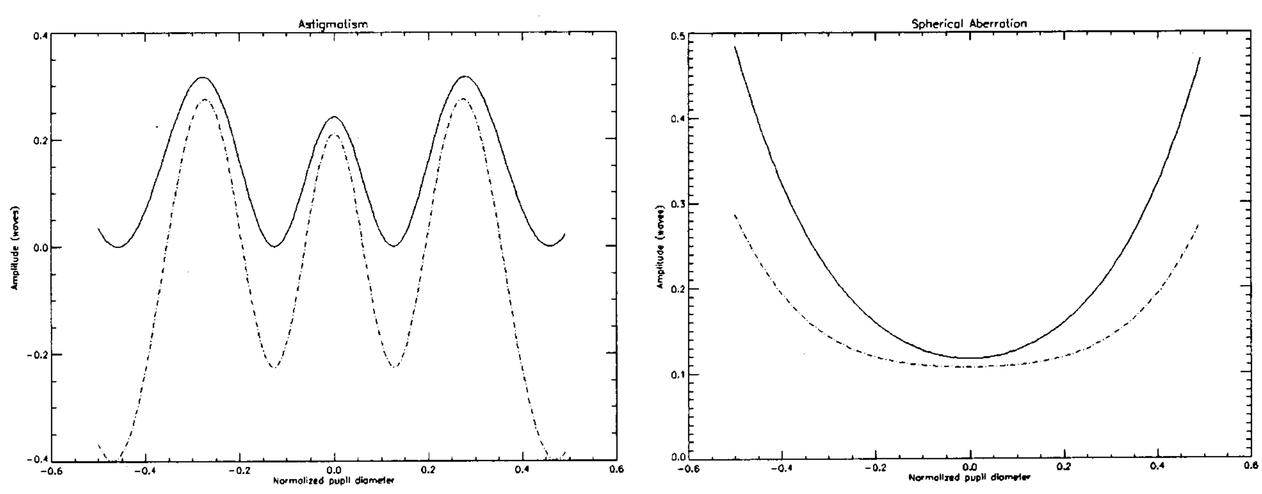

Fig. 6 Cross-cuts of the Zernike mode for astigmatism (left) and spherical aberration (right) for filled pupil (solid line) and annular pupil (dashed line).

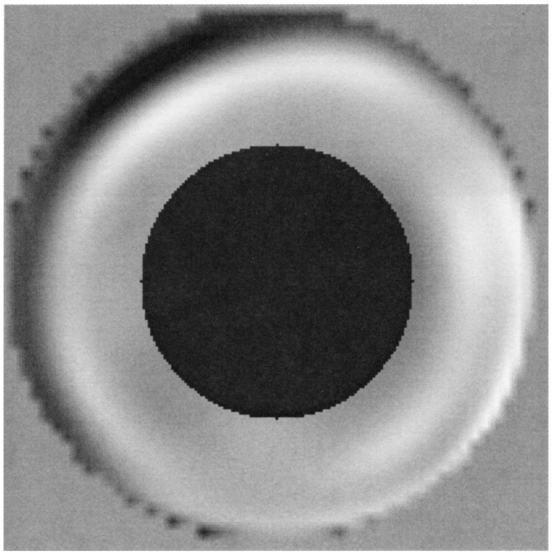

Fig. 7 Reconstruction of the wavefront using a 40-term expansion of the annular pupil Zernike fit of the data.

telescope was due to the large secondary obscuration and the effect that this may have on the number of illuminated actuators in the pupil available for correction. Since many of the active elements of the wavefront corrector would be hidden in the shadow of the secondary and thus not illuminated and available for use, the effectiveness of the wavefront corrector could be significantly reduced.

4 Summary

A phase diversity analysis of the NOFS 1-m telescope was carried out. The results show that the rms wavefront error, which we have assumed as a static error in the optics, is small and easily correctable using a wide variety of standard wavefront-correcting elements. Using such optical elements to correct the static aberrations will leave sufficient residual throw in the deformable element to compensate for atmospheric seeing.

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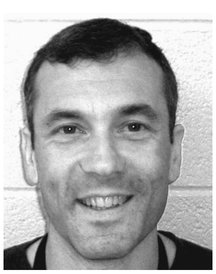

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