Seismo-Acoustics in Laterally Varying Media

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Introduction: Many wave propagation problems in the geosciences involve a nearly stratified waveguide. When lateral variations in the medium (ocean, solid Earth, or atmosphere) are sufficiently gradual, outgoing energy dominates backscattered energy, and solutions can be obtained using an approximate wave equation that only accounts for outgoing waves.[1] This approach, which is known as the parabolic equation method, often improves efficiency by several orders of magnitude with no significant loss in accuracy. For example, outgoing solutions can be obtained in minutes for global-scale ocean acoustics problems[2] that would be out of the question to solve in terms of the full-wave equation.

Improved Techniques: The extension of the parabolic equation method to problems involving solid layers has been an area of great interest for nearly 30 years. Solid layers are governed by the elastic-wave equation and support two types of body waves and interface waves. This vector equation is more complicated than the scalar equation that governs acoustic waves, and the solutions are more prone to instabilities. Due to these difficulties, it was nearly 20 years between the introduction of the parabolic equation method into ocean acoustics and its first successful application to problems involving elastic layers.[3]

For the past decade, there has been a focus on improving the accuracy of parabolic equation solutions when there are lateral variations in solid layers. One of the key developments in this area was to rederive the elastic parabolic equation in terms of a new set of dependent variables.[4] In order to facilitate the derivation of the parabolic-wave equation from the full-wave equation, it is necessary to choose the dependent variables so that they have certain symmetry properties. Otherwise, it is impossible to derive a parabolic-wave equation. The first derivations were based on an unconventional choice of variables. The fact that one of the variables is not continuous makes it difficult to handle interfaces between solid layers. The new formulation is also based on an unconventional choice of variables, but both of the variables are continuous, and this has made it possible not only to handle arbitrary layering, but also to handle lateral variations more accurately.

We have considered two approaches for handling lateral variations, and both of them have produced promising results. One of the approaches is based on rotating coordinates so that one of the coordinates is aligned with the interface. This approach has proven to be effective for handling a sloping ocean bottom, which is an important and difficult case. Appearing in Fig. 6 is a problem involving two types of interface waves that was solved with this approach. In this example, a Scholte wave propagates along the ocean bottom and onto the beach, where it becomes a Rayleigh wave. We are also developing approaches based

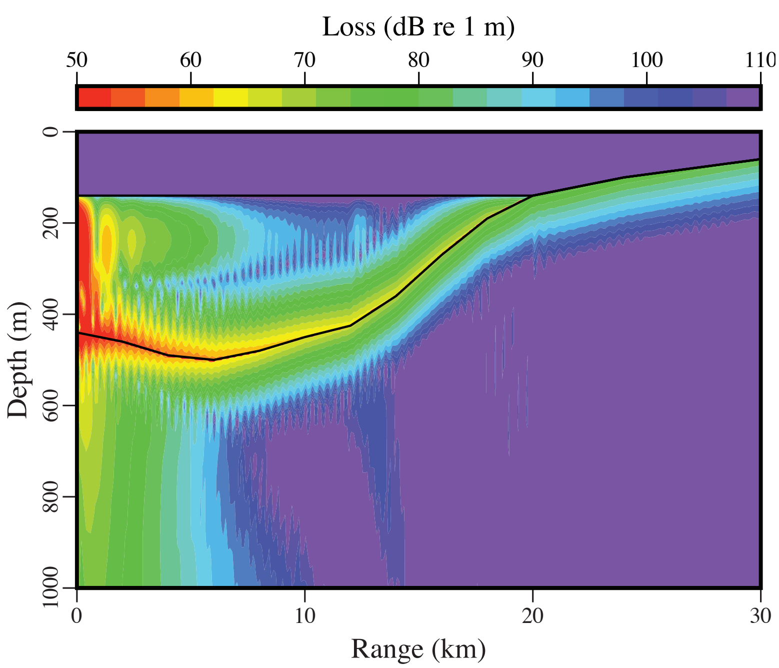

FIGURE 6
Propagation of a Scholte wave along the ocean bottom. This interface wave becomes a Rayleigh wave beyond the beach.

on the original coordinate system that can handle a larger class of lateral variations. These approaches are based on making corrections to the field as the properties of the waveguide vary with range. Appearing in Fig. 7 is a seismic problem involving complex layering that was solved with this approach.

Comparison with Data: When possible, we compare parabolic equation solutions with solutions obtained by other means. Since such solutions are very limited, we have also made comparisons with data from tank experiments. Appearing in Fig. 8 is a comparison of the parabolic equation solution with data taken in a carefully controlled experiment involving a slab of poly vinyl chloride (PVC) that was suspended in a tank of water to model a sloping, elastic ocean bottom (Fig. 9).

[Sponsored by ONR]

References

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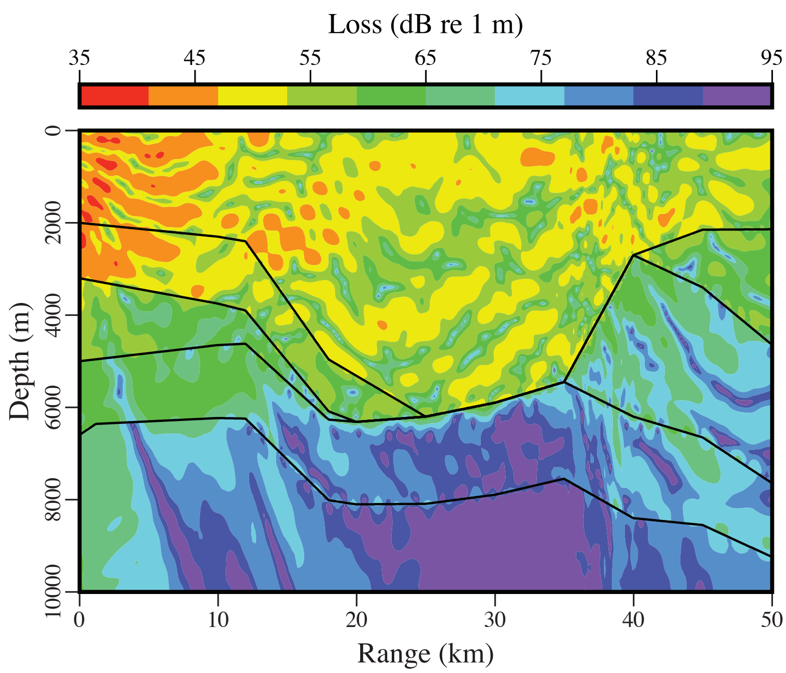

FIGURE 7
Elastic wave propagation in a medium with complex layering. This plot only shows the compressional wave energy.

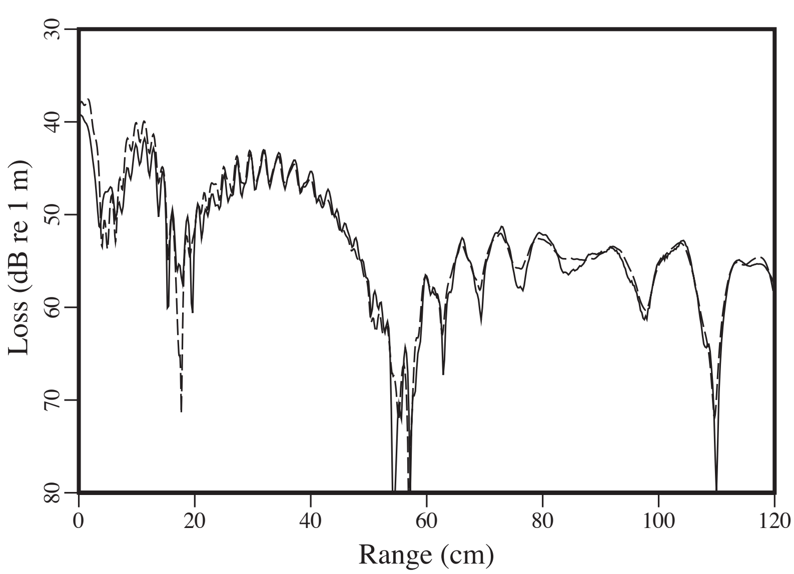

FIGURE 8
Comparison between model results (dashed curve) and tank data (solid curve) along a line at constant depth. The frequency is 225 kHz.

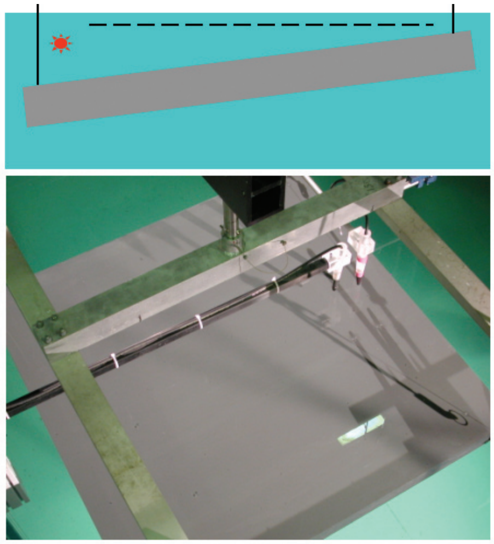

FIGURE 9
Geometry of the tank experiment.

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