Hierarchical Planning: Relating Task and Goal Decomposition with Task Sharing

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Abstract

Considerable work has focused on enhancing the semantics of Hierarchical Task Networks (HTNs) in order to advance the state-of-the-art in hierarchical planning. For instance, the Hierarchical Goal Network (HGN) formalism operates over a hierarchy of goals to facilitate tighter integration of decompositional planning with classical planning. Another example is the Action Notation Markup Language (ANML) which adds aspects of generative planning and task-sharing to the standard HTN semantics.

The aim of this work is to formally analyze the effects of these modifications to HTN semantics on the computational complexity and expressivity of HTN planning. To facilitate analysis, we unify goal and task planning into Goal-Task Network (GTN) planning. GTN models use HTN and HGN constructs, but have a solution-preserving mapping back to HTN planning. We then show theoretical results that provide new insights into both the expressivity as well as computational complexity of GTN planning under a number of different semantics. Our work lays a firm footing to clarify exact semantics for recent planners based on ANML, HGNs, and similar hierarchical languages.

1 Introduction

Hierarchical Task Network (HTN) planning [Erol et al., 1994] is a task planning formalism that is widely used in real-world applications [Nau et al., 2005]. HTN planners use recipes to recursively decompose tasks into primitive action sequences. We focus on two hierarchical formalisms, the Action Notation Markup Language (ANML) [Smith et al., 2008] and Hierarchical Goal Network (HGN) planning [Shivashankar et al., 2012], and their relationship with the more widely known HTN planning. All these formalisms now have systems that implement subsets of their features, yet little research examines the differences between them, either in terms of their computational complexity or their semantics.

HGN planning bridges classical planning and totally ordered task networks by operating over sequences of goals with methods that decompose goals with further subgoals. By attaching goals to methods, HGN planning adds explicit semantics to methods, making it easy to adapt classical heuristics to HGN planning [Shivashankar et al., 2016].

The Action Notation Markup Language (ANML) is a temporal planning framework with a hierarchical component [Smith et al., 2008], designed to address common challenges faced when modeling problems that require sophisticated temporal or resource reasoning within a single language that combines generative and hierarchical task planning [Smith and Cushing, 2008]; an essential feature of ANML is its ability to embed complex procedures as tasks. A translation from ANML to a variant of PDDL [Bonasso et al., 2009] and the planning system FAPE [Dvorak et al., 2014] have implemented subsets of ANML.

One of the unique features of ANML is that it allows sub-tasks to be shared between multiple tasks. For example, let us assume an ANML planner is considering two tasks t₁ and t₂ that are unordered with respect to one another which decompose into the same setup task; so, t₁ decomposes into \{setup, a₁\} (read "do setup, then do a₁"), while t₂ decomposes into \{setup, a₂\}. According to standard HTN planning semantics, the only solution would involve two setup tasks. ANML, however, also allows for sharing the same setup task across both task-decompositions, and can therefore generate a solution with a single setup task.

Figure 1: Summary of plan-preserving translations between problem classes. An edge \((X, Y)\) indicates problem instances under \(X\)’s semantics can be translated to problem instances under \(Y\)’s semantics while preserving the solution set. Subscripts indicate task insertion (I) or sharing (S) semantics. Solid lines indicate translations within complexity classes and dotted lines indicate translations across complexity classes. With the exception of the two HTN results, all results are new.
Traditional HTN planners strictly follow the method hierarchy when composing a plan. This rigidity is a liability in dynamic environments when actions may fail and the plan needs to be repaired [Ayan et al., 2007]. Task Insertion semantics modifies HTN semantics to allow planners to insert actions whenever they are useful, regardless of methods [Geier and Bercher, 2011]. Task insertion is also used in the HTN literature to capture plan repair and partial method knowledge. These semantics are also implemented as part of HGNs [Shivashankar et al., 2013] and are explicitly supported in ANML. It is not known how task insertion interacts with task sharing.

**Contributions.** The central aim of this paper is to unify these various families of hierarchical planning and their semantics as summarized in Figure 1. In particular,

- **A Unified Formalism:** We define Goal-Task Network (GTN) planning that models partially ordered HTN planning and extends HGN planning with partially-ordered sets of subgoals. We present standard semantics for GTN that extends HTN semantics with goals. We then extend these semantics with the addition of task insertion semantics, denoted GTN₁, with the addition of task sharing GTN_S, and finally, with the combined semantics of GTN₁S.

- **Expressivity Results:** We show that every GTN problem with any combination of the above semantics can be translated in a solution preserving manner to HTN planning with standard semantics, which is in general semi-decidable.

- **Membership Results:** For certain classes of GTN planning, we improve the semi-decidable upper bound by proving that HGN₁ is in PSPACE, while GTN₁S and GTN₁ planning are in NEXPTIME.

2 A Unified Hierarchical Planning Formalism

To precisely describe the relationship between HTN and HGN planning, we model both in a single framework called Goal-Task Network (GTN) planning¹. GTN planning has already been extremely useful in formalizing goal reasoning [Roberts et al., 2016]. We augment the set-theoretic notation of Geier and Bercher [2011] with goal decomposition from HGN planning [Shivashankar et al., 2012] and with SHOP2-style method preconditions [Nau et al., 2003]. After formalizing GTNs, we clarify their operations before discussing GTN problems and their solutions.

2.1 Preliminaries

Let \( \mathcal{L} \) be a propositional language and \( \mathcal{T} \) be a set of task names represented as propositional symbols not appearing in \( \mathcal{L} \) with \( \mathcal{L} \cap \mathcal{T} = \emptyset \), and let \( O \) and \( C \) be a partition of \( \mathcal{T} \) (\( O \cup C = \mathcal{T}, \mathcal{O} \cap \mathcal{C} = \emptyset \)). \( O \) will correspond to primitive tasks, or actions that can be executed directly, while \( C \) represents Compound or non-primitive tasks which need to be recursively decomposed into primitive tasks before they can be executed.

Then HTN planning is over partially-ordered multisets of task names in \( \mathcal{T} \) representing activities to accomplish, while HGN planning is over totally-ordered subgoals in \( \mathcal{L} \) which must be met in sequence. GTNs elegantly models both, and in addition extends the scope of HGN planning to allow for partially-ordered sets of subgoals.

2.2 Goal-Task Networks (GTNs)

A goal-task network is a tuple \((I, \prec, \alpha)\) where \( I \) is a set of instance symbols which are placeholders for task names and goals, \( \prec \subset I \times I \) is a partial order on \( I \), and \( \alpha : I \rightarrow \mathcal{L} \cup \mathcal{T} \) maps each instance symbols to a goal or task name.

Some symbol instances are special because they occur first or last in a network. An instance symbol \( i \) is unconstrained if no symbols are constrained before it (\( \forall i' \in I, i' \neq i \)) and last if it constrained after all other symbols (\( \forall i' \in I, i' < i \)). We refer to \( i \) as a task if \( \alpha(i) \in \mathcal{T} \) and as a subgoal if \( \alpha(i) \in \mathcal{L} \); recall that \( \mathcal{L} \) and \( \mathcal{T} \) are disjoint.

Two goal-task networks, \((I_1, \prec_1, \alpha_1)\) and \((I_2, \prec_2, \alpha_2)\) are isomorphic if there exists a bijection \( f : I_1 \rightarrow I_2 \) such that: (1) \((i, i') \in \prec_1 \iff (f(i), f(i')) \in \prec_2 \) and (2) \( \alpha_1(i) = \alpha_2(f(i)) \) for all \( i \in I_1 \).

**Methods.** We distinguish the methods for GTNs by the kind of symbol they decompose. A task method \( m_t \) is a tuple \((n, \chi, gtn)\) where \( n \in C \) is a non-primitive task name, \( \chi \in \mathcal{L} \) is \( m_t \)'s precondition, and \( gtn \) is a goal-task network over \( \mathcal{L} \) and \( \mathcal{T} \). A task method is relevant to a task \( i \) in a goal-task network \((I, \prec, \alpha)\) if \( n = \alpha(i) \). The method \( m_t \) is a specific decomposition of a task \( n \) into a partially-ordered set of subtasks and subgoals, and there may be many such methods.

A goal method \( m_g \), similarly, is a tuple \((g, \chi, gtn)\) where \( g, \chi \in \mathcal{L} \) are the goal and precondition of \( m_g \) and \( gtn \) is a goal-task network. A goal method is relevant to a subgoal \( i \) in a goal-task network \((I, \prec, \alpha)\) if at least one literal in the negation-normal form (NNF) of \( g \) matches a literal in the NNF of \( \alpha(i) \) (e.g., accomplishing \( g \) ensures that at least part of \( \alpha(i) \) is true). By convention, the goal-task network \( gtn = ((I, \prec, \alpha) \chi g) \) has a last instance symbol \( i \in I \) with \( \alpha(i) = g \) to ensure that \( m_g \) accomplishes its own goal.

Task and goal methods are respectively identical to methods in HTN and HGN planning.

**Operators.** An operator \( o \) is a tuple \((n, \chi, e)\) where \( n \in O \) is a primitive task name (assumed unique to \( o \)), \( \chi \) is a propositional formula in \( \mathcal{L} \) called \( o \)'s precondition (or \( prec(o) \)), and \( e \) is a set of literals from \( \mathcal{L} \) called \( o \)'s effects. We refer to the set of positive literals in \( e \) as \( add(o) \) and the negated literals as \( del(o) \). An operator is relevant to primitive task \( i \) if \( n = \alpha(i) \) and to a subgoal \( g \) if the effects of \( o \) contain a matching literal from the NNF of \( \alpha(i) \).

**States.** A state \( s \in \mathcal{L}_{sym} \) is any subset of \( \mathcal{L}_{sym} \), the propositional symbols of \( \mathcal{L} \); the set of all states is \( 2^{\mathcal{L}_{sym}} \). A set of operators \( \mathcal{O} \) forms a transition (partial) function \( \gamma : 2^{\mathcal{L}_{sym}} \times \mathcal{O} \rightarrow 2^{\mathcal{L}_{sym}} \) as follows: \( \gamma(s, o) \) is defined if \( s \models prec(o) \) (the precondition of \( o \) holds in \( s \)), and \( \gamma(s, o) = (s \setminus del(o)) \cup add(o) \).
2.3 GTN Nodes and Node Operations

A node is a (state, goal-task network) pair \( N = (s, gttn) \). GTN planning allows four operations on nodes, described below.

We use the term progression as an umbrella term for the various GTN node operations. We write \( N \rightarrow_P N' \) if any of the operations can make that transition between nodes. For progression sequences from \( N \) to \( N'' \), we write \( N \rightarrow_P, N'' \).

Operator application of an operator \( o \) to a node \( (s, gttn) \), with \( gttn = (I, \prec, \alpha) \), written as \( (s, gttn) \xrightarrow{\iota} \alpha \) \( (s', gttn') \), is defined if \( s \models prec(o) \) and \( o \) is relevant to an unconstrained instance symbol \( i \) in \( gttn \). If \( i \) is a primitive task with task name \( n \), then this corresponds to primitive task application in HTNS; the result of application is given by \( s' = \gamma(s, o) \) and \( gttn' = (I \setminus \{i\}, \{(i_1, i_2) \in \prec | i_1 \neq i\}, \{(i', n) \in \alpha | i' \neq i\}) \), transitioning the state and removing the instance symbol from the network. If \( i \) is a relevant goal task instead, this corresponds to primitive task application in HGNs; in this case, \( gttn' = gttn \), and the subgoal remains while the state changes.

Goal release for an unconstrained subgoal \( i \), written \( (s, gttn) \xrightarrow{G} \alpha \) \( (s', gttn') \), is defined whenever \( s \models \alpha(i_j) \). Goal release can remove a subgoal whenever it is satisfied by \( s \) and so \( gttn' \) is given by \( (I \setminus \{i\}, \{(i_1, i_2) \in \prec | i_1 \neq i\}, \{(i', n) \in \alpha | i' \neq i\}) \), just as in operator application.

Task decomposition for an unconstrained task \( i \) by a relevant task method \( m = (c, X, gttn_m) \), written \( (s, gttn) \xrightarrow{m} \alpha \) \( (s', gttn') \), is defined whenever \( s \models \chi \). Task decomposition expands \( i \) in \( gttn \), replacing it with the network \( gttn_m \). Let \( (I, \prec, \alpha) = gttn_m \) and \( (I_m, \prec_m, \alpha_m) = gttn_m \), assuming without loss of generality that \( I \cap I_m = \emptyset \). Then the result of task decomposition, \( gttn' = (I', \prec', \alpha') \) is given by:

\[
\begin{align*}
I' & := (I \setminus \{i\}) \cup I_m \\
\prec' & := \{(i, i') \in \prec | i, i' \in I'\} \cup \prec_m \\
\cup & \{(i_1, i_2) \in I_m \times I | (i_2, i) \in \prec\} \\
\alpha' & := \{(i, n) \in \alpha | i \in I'\} \cup \alpha_m.
\end{align*}
\]

Goal composition for an unconstrained subgoal \( i \) by a relevant goal method \( m = (g_m, \chi, gttn_m) \), also written \( (s, gttn) \xrightarrow{m} \alpha \) \( (s', gttn') \), is defined whenever \( s \models \chi \). Goal composition prepends \( i \) with \( gttn_m \), leaving the subgoal in place, so that \( gttn' \) is given by \( (I \cup I_m, \prec \cup \prec_m \cup \{(i, i_m) \in I_m \times \{i\}, \alpha \cup \alpha_m\}) \).

3 GTN Planning Problems and Solutions

A problem is a tuple \( P = (L, O, M, N_0) \) where \( L \) is propositional language for defining the set of operators \( O \), the set of goal and task methods \( M \), and \( N_0 \) is the initial node consisting of the initial goal-task network \( gttn_0 \), and the initial state \( s_0 \). \( O \) and \( C \) are implicitly defined by \( O \) and \( M \). Nodes are isomorphic if their goal-task networks are isomorphic and their states are identical.

We say \( P \) is solvable under GTN semantics iff there is a progression \( N_0 \rightarrow_P N_k \), where \( N_k = (s_k, gttn_k) \), \( s_k \) is any state, \( gttn_k \) is the empty network. The subsequence of operator applications of that progression is a plan for \( P \).

Whenever all networks (method and initial) in a problem \( P \) contain only tasks, we call it an HTN problem since goal release and goal decomposition are irrelevant, and so the semantics are identical to HTN planning under progression [Alford et al., 2012]. Whenever all networks contain only subgoals, we call it an HGN problem, since the semantics are identical to HGN planning with a straightforward extension to partially-ordered networks [Shivashankar et al., 2012].

3.1 Task-Insertion Semantics

Formally, task insertion of an operator \( o \) is defined whenever \( o \)’s precondition is supported by the state, and is written \((s, gttn) \xrightarrow{\iota} (s', gttn'), \) where \( s' = \gamma(s, o) \). Note that this combines two operations, insertion of \( o \) and operator application of \( o \).

If a node \( (s', gttn') \) is reachable from another node \((s, gttn)\) via any sequence of progressions and insertions, we write \((s, gttn) \rightarrow_P (s', gttn') \). We say that a problem \( P \) is solvable under GTN semantics with task insertion (GTN\(_I\) semantics) iff \((s_0, gttn_0) \rightarrow_P (s, gttn) \) where \( s \) is any state and \( gttn_0 \) is the empty goal-task network. HTN\(_I\) planning is NEXPTIME-complete [Alford et al., 2015b].

3.2 Task-Sharing Semantics

We formalize task sharing here using a task merging operation that takes two task symbols in the goal-task network that map to the same task name and merges them into one task symbol. Formally, task merging of two unconstrained tasks \( i_1, i_2 \) in a network \( gttn = (I, \prec, \alpha) \) is applicable whenever the tasks share a task name \( (\alpha(i_1) = \alpha(i_2)) \). The operation, written \((s, gttn) \xrightarrow{i_1,i_2} \alpha \) \( (s', gttn') \), is given by \( gttn' = (I \setminus \{i_2\}, \prec', \alpha') \), where:

\[
\prec' := \{(i, i') \in \prec | i \neq i'\} \cup \{(i_1, i_2) \in I_m \times I | (i_2, i) \in \prec\} \cup \{(i_1, i_2) \in I_m \times I | (i_2, i) \in \prec\}
\]

We say a problem \( P \) with initial node \((s, gttn)\) is solvable under GTN semantics with task sharing (GTN\(_S\) semantics) if there is any sequence of progression and merging to a node \((s', gttn')\), written \((s, gttn) \rightarrow_P (s', gttn') \). Solvability under both task insertion and sharing (GTN\(_I_S\) semantics) is defined similarly, with the sequence denoted as \((s, gttn) \rightarrow_P (s', gttn') \).

4 Plan-preserving translations

We can study the computational complexity of GTN planning by reductions, which are transformations between problems and/or formalisms. In this section, we use a special kind of reduction we call plan-preserving translations, which preserve the solution set of the problem across translation. For example, we can trivially use a GTN planner to solve a GTN\(_I\) problem \( P \) by adding a new task to the initial node along with methods that decompose it into any sequence of operators. Since this transformation preserves the set of plans, (1) an optimal plan from the GTN planner is an optimal plan \( P \), and (2) when creating GTN\(_I\) planners, the transformation is a procedure for using any admissible GTN heuristic as a GTN\(_I\) heuristic. Further, we know that GTN planning is at least as
expressive as GTN₁ planning, since the (possibly infinite) set of solutions for any problem under GTN₁ semantics can be exactly expressed as the solutions of the translated problem under GTN semantics.

**Same-semantic translations from GTNs to HTNs**

In this section, we provide a plan preserving translation from GTN problems to HTN problems that preserves plans under like semantics (GTN to HTN, GTN₁ to HTN with task insertion, etc.). This establishes that GTN planning is no harder in the general case than HTN planning across all the semantic variants we discuss.

**Construction 4.1.** Let \( P \) be a problem. We iteratively construct an HTN problem \( P' \) as follows:

Let \( g \) be a subgoal appearing in the initial network or any method network. Let \( t_g \) be a fresh task name, and let \( m_g = (t_g, g, gt_{n_g}) \) which will act as the goal release operation for \( t_g \). Let \( N'_0 \) and \( M' \) be the \( P \)'s initial node and set of methods where each instance of \( g \) in a network is replaced by \( t_g \). Let \( M_O \) be a set methods of the form \( m_{o} = (t_g, \chi, gt_{n_{o}}) \) for each operator \( o \) relevant to \( g \), where \( gt_{n_{o}} \) contains the task \( o \) preceding the task \( t_g \). Finally, let \( M_{Og} \) be a set methods of the form \( m_{og} = (t_g, \chi, gt_{n_{og}}) \) for each goal method \( (g', \chi, gt_{n'}) \in M' \) relevant to \( g \), where \( gt_{n_{og}} \) is the new task for \( t_g \) constrained to come after all other instance methods. Then \( P' = (L, O, M' \cup M_O \cup M_{Og} \cup \{ m_{og} \}; N'_0) \). Iterate until no goal appears in a network, and remove all goal methods.

**Theorem 4.2.** For a problem \( P \) with initial node \( N_0 \), use Construction 4.1 to construct an HTN problem \( P' \). Then, under GTN semantics, a sequence of actions is a plan for \( P \) if it is a plan for \( P' \). The same result holds under GTN₁ semantics.

**Proof.** We prove this theorem by showing that for each iteration of Construction 4.1, after replacing \( g \) with \( t_g \), \( P \) and \( P' \) have the same set of plans.

\( \implies \) Let \( N_0 \rightarrow_P N_k \) be a derivation of a plan \( P \) for \( P' \), where \( N_0 \sim \cdots \sim N_{k-1} \). Construct \( N'_0 \rightarrow_{P'} N'_k \) as follows: \( N'_0 \) is isomorphic to \( N_0 \) with \( g \) replaced with \( t_g \). Suppose \( N'_0 \) is isomorphic to \( N_0 \) with \( g \) replaced with \( t_g \). The following cases ensure the same for \( N'_1 \) and \( N'_{k+1} \):

If \( N_i \xrightarrow{i \circ A} N_{i+1} \) is an application of \( o \) to goal task \( g \), then decompose the corresponding \( i \) in \( N'_i \) with \( m_{og} \) and immediately apply \( o \) in \( N'_{i+1} \). If \( N_i \xrightarrow{i \circ G} N_{i+1} \), then \( g \) is satisfied in \( N_i \)'s state (identical to the state of \( N'_i \)), so decompose the corresponding \( i \) from \( N'_i \) with \( m_{og} \), removing \( i \) to get the corresponding \( N'_{i+1} \). If \( N_i \xrightarrow{i \circ M} N_{i+1} \) corresponds to goal-method decomposition, then \( N'_i \xrightarrow{i \circ m_{og}} N'_{i+1} \) is applicable and produces the corresponding \( N'_{i+1} \). Otherwise, the derivation does not involve \( g \), so apply it directly to \( N'_i \). Since this preserves operator applications in the derivation, \( p \) is also a plan for \( P' \).

\( \impliedby \) Let \( N'_0 \rightarrow_{P'} N'_k \) be a derivation of a plan \( P' \). Note that decomposition with any \( m_{og} \) can be pushed back in the derivation until it is immediately before the application of \( o \). Then the inverse of the above procedure can produce a derivation of \( p \) for \( P \).

Since the set of GTN problems includes all HTN problems and HTN planning is semi-decidable [Erol et al., 1996], by the above theorem, GTN planning is also semi-decidable:

**Corollary 4.3.** Deciding whether a solution exists for a problem under GTN semantics is semi-decidable.

We know by Alford et al. [2015b] that HTN planning with task insertion is NEXPTIME-complete, so by the above, the same result extends to GTN₁ planning:

**Corollary 4.4.** Deciding whether a solution exists for a problem under GTN₁ semantics is NEXPTIME-complete.

We can also show that Construction 4.1 preserves plan sets under GTN₅ semantics:

**Theorem 4.5.** For a problem \( P \), let \( P' \) be the resulting HTN problem from Construction 4.1. Then, under GTN₅ semantics, a sequence of actions is a plan for \( P \) if it is a plan for \( P' \). The same result holds under GTN₁₅ semantics.

**Proof.** Again, we show that for each iteration of Construction 4.1, after removing \( g \), \( P \) and \( P' \) have the same set of plans.

\( \implies \) Task merging does not involve goals, so given a derivation \( N_0 \rightarrow_P^* N_k \) of a plan \( P \) for \( P' \), extend the \( (\implies) \) procedure of Theorem 4.2 with merging so that \( N_j \xrightarrow{i_1 \sim i_2} N_{j+1} \) maps to \( N'_j \xrightarrow{i_1 \sim i_2} N'_{j+1} \).

\( \impliedby \) Once \( g \) is replaced by \( t_g \), there are two threats to inverting the above procedure to find a derivation of a plan \( P \) for \( P' \): (1) two tasks for \( t_g \) may be merged, and (2) a primitive task for an operator \( o \) may be merged with another task for \( o \).

For (1), if there is a merge \( N_j \xrightarrow{i_1 \sim i_2} N_{j+1} \) where \( \alpha(i_1) = t_g \) in the derivation of \( p \), then there must also be an \( N'_q \) \((q > j)\) where the method corresponding to goal release is applied (meaning \( g \) holds in the state of \( N'_q \)). Then, since \( i_1 \) and \( i_2 \) are not ordered with respect to each other, another derivation of \( p \) is to skip the merge of \( i_2 \), and decompose \( i_2 \) with \( m_{og} \) immediately after \( N_q \), since \( g \) must hold in that node’s state.

For (2), assume WLOG that the task \( i_2 \) \((\alpha(i_2) = o)\) was inserted via decomposition with \( m_{og} \) of a task \( i_3 \). The derivation of \( p \) must include a decomposition of \( i_3 \) with \( m_{og} \), which replaces \( i_3 \)'s task for \( t_g \) with the tasks \( o \) preceding \( t_g \). Later in the derivation, the task for \( o \) is merged with another task, and later applied, all before any further decomposition of \( t_g \). Then another derivation of \( p \) is to just skip the first decomposition of \( i_3 \) and subsequent merge.

Since we can eliminate any merging of \( t_g \) or its related primitive tasks, we can invert \( (\implies) \) to find a derivation of \( p \) for \( P \) from a derivation of \( P' \).

\( \square \)

As a special case, consider that HGN problems have the same plan set under GTN₁₅ semantics as under GTN semantics, as there are no tasks to merge. So by the above theorem, Construction 4.1 is a plan-preserving translation of HGN problems to HTN₅ planning:
Corollary 4.6. For any HGN problem $P$, let $P'$ be the resulting HTN problem from Construction 4.1. Then a sequence of actions is a plan for $P$ under GTN semantics iff it is a plan for $P'$ under GTN$_S$ semantics. The same result holds under GTN$_{TS}$ semantics.

So HGN planning is neither harder nor more expressive than HTN planning with sharing.

Translating from GTN$_S$ to GTN

In this section, we give a plan preserving translation from GTN$_S$ semantics to standard GTN semantics as part of our proof that GTN$_S$ planning is semi-decidable. To start, we note that, at least as far as solvability goes, GTN$_S$ semantics are equivalent to a sharing semantics that only allows merging of primitive operators.

Lemma 4.7. Let $N_0 \rightarrow_{\mathcal{P}M}^1 N_k$ be the derivation of a plan $p$ for problem $P$. Then there is another derivation $N_0 \rightarrow_{\mathcal{P}M}^1 N_k$ that any merging only happens for primitive tasks.

Proof. We can iteratively remove merging of non-primitive tasks from $N_0 \rightarrow_{\mathcal{P}M}^1 N_k$ as follows: Let $N_j \rightarrow_{\mathcal{P}M}^i N_{j+1}$ be the first merge of any two non-primitive tasks $i_1, i_2$ in the derivation $p$. Since $i_1$ is a non-primitive task, there is a subsequent decomposition $N_k \rightarrow_{\mathcal{D}M}^i N_{k+1}$. Let $N_r \rightarrow_{\mathcal{P}M}^{i_1, m} N_{r+1}$ be the same sequence of progressions above with the initial merge omitted (including the decomposition of $i_1$). Then decompose $i_2$ with the same method as $i_1$ and merge all its resulting tasks with those from the decomposition of $i_1$. The resulting node is isomorphic with $N_{k+1}$.

We can now reduce GTN$_S$ planning to GTN relying on operators asserting in the state when they have been applied:

Construction 4.8. Let $P = (\mathcal{L}, \mathcal{O}, \mathcal{M}, N_0)$ be a GTN problem. We iteratively construct a new GTN problem $P'$ as follows: For each primitive task name $o$, we introduce a new non-primitive task symbol $t_o$, as well as a new propositional symbol $f \langle o \rangle$. Let $\mathcal{L}' = \mathcal{L} \cup \{f \langle o \rangle | o \in \mathcal{O}\}$. Let $\mathcal{O}'$ be the set of operators of $\mathcal{O}$ modified so that each operator $o$ asserts $f \langle o \rangle$ and retracts $f \langle o \rangle$ for all $o' \neq o$. Let $N'_0$ and $\mathcal{M}'$ be $N_0$ and $\mathcal{M}$ with all primitive tasks in their networks replaced with the corresponding $t_o$. Finally, let $\mathcal{M}'$ contain two methods for each operator $o$: $m_{\langle o \rangle} = (t_o, true, \text{gt} \langle o \rangle)$, where $\text{gt} \langle o \rangle$ contains only the task $t_o$, and $m_{\langle o \rangle} = (t_o, f \langle o \rangle, \text{gt} \langle o \rangle)$, which can remove $t_o$ from a network immediately after $o$ is applied. Then $P' = (\mathcal{L}', \mathcal{O}', \mathcal{M}' \cup \mathcal{M}, N'_0)$.

There are two important points to note in the above construction. First, $f \langle o \rangle$ is set up in such a way that it is true iff $o$ was the last operator applied. Second, we use $f \langle o \rangle$ to simulate task-sharing under GTN semantics by providing two methods $m_{\langle o \rangle}$ and $m_{\langle o \rangle}$ for each $o$. These respectively introduce and don’t introduce a new instance of $o$ after one instance of $o$ has already been applied; $m_{\langle o \rangle}$ thus simulates the task-sharing option. Below, we formally prove that this construction indeed simulates task-sharing under GTN semantics:

Theorem 4.9. Let $P'$ be the result of applying Construction 4.8 to a problem $P$. Then a sequence of operators $p$ is a plan for $P$ under GTN semantics iff it is a plan for $P'$ under GTN semantics. This also holds for $P$ under GTN$_{TS}$ semantics and $P'$ under GTN$_S$ semantics.

Proof. $(\Rightarrow)$ By Lemma 4.7, there is a derivation of $P$ under GTN$_S$ semantics that only merges primitive operators. We can also assume WLOG that merging comes immediately before operator application. For a derivation of $p$ from $P'$ under GTN semantics, replace each sequence of merges then application of an operator $o$ with decomposition by $m_{\langle o \rangle}$, application of $o$, and decomposition with $m_{\langle o \rangle}$ for the remaining merges.

$(\Leftarrow)$ Since $m_{\langle o \rangle}$ has no precondition, we can always move it back in a derivation of $p$ until just before the application of $o$. Similarly, we can move forward any $m_{\langle o \rangle}$ before any intervening decompositions until they come just after application of $o$. Then invert $(\Rightarrow)$ to obtain the appropriate merging.

The only threat to this step under task insertion is that there is an insertion of $o$ followed by applications of $m_{\langle o \rangle}$. If this happens, then there must be at least one unconstrained task for $o$ in the current network. We can replace the insertion and first $m_{\langle o \rangle}$ with decomposition by $m_{\langle o \rangle}$ and $o$’s application.

Since we have a plan-preserving polynomial translation of GTN$_S$ planning into HTN planning, GTN$_S$ planning is not computationally harder nor more expressive than HTN planning.

5 Encoding HTNs as HGNs

Although there are a wide array of complexity classes for specific restrictions of HTN planning [Alford et al., 2015a], HTN planning in general is semi-decidable [Erol et al., 1996]. Erol et al.’s proof of undecidability encodes the intersection of context free grammars (CFGs) as an HTN problem with totally-ordered methods and two partially-ordered initial tasks. The two tasks function as producer and consumer - one producing a sequence of symbols from one CFG, and the other consuming them in the order of another. The reachable tasks from both initial tasks are disjoint. Thus, any progression of the initial problem leads to a task network composed of two totally-ordered sets of tasks. Under GTN$_S$ semantics, no task within one of the sets can be merged (they are all ordered with respect to each other), and no task between sets can be merged (the two sets have distinct task names). Thus, Erol et al.’s construction has the same set of solutions under GTN semantics as GTN$_S$ semantics.

For HGNs, Shivashankar et al. [2012] gave a bidirectional encoding of totally-ordered HGNs and HTNs, encoding task names as new goals. This easily extends to encode Erol et al.’s construction in partially-ordered HGN planning:

Theorem 5.1. Deciding whether a solution to an HGN or GTN$_S$ problem exists under GTN$_S$ semantics is semi-decidable.

These results do not easily extend to task insertion with sharing. In particular, the proof that HTN$_S$ planning is NEXPTIME-hard encodes a Turing machine into the task network where an exponential number of partially-ordered tasks are synchronized through the state. In the following lemma and construction, we show that if there is a plan where two mergeable tasks were not merged, we can always merge the
two tasks and replace the lost task’s primitive descendants with task insertion.

In a derivation \( N_0 \rightarrow_{p, N_k} \), we say that an instance symbol \( i' \) in a node \( N_j \) is a child of a symbol \( i \) in node \( N_{j-1} \) iff (1) \( i = i' \) and was not otherwise involved in the progression or (2) \( i' \) was introduced in a decomposition \( N_{j-1} \xrightarrow{i,m,D} N_j \). A descendant symbol of \( i \) is any child in the subsequent node, or a descendant of those children in following nodes, while the reverse holds for ancestors.

**Lemma 5.2.** Let \( N_0 \rightarrow_{p_{IM}} N_k \) be a derivation of a plan \( p \) for a problem \( P \) under GTN_{15} semantics, and let \( N_j \) be a node in that sequence with two mergeable tasks \( i_1 \) and \( i_2 \). Then there is a progression \( N_j' \rightarrow_{p_{IM}} N_k \) which preserves the remain sequence of operator application after merging \( i_1 \) and \( i_2 \).

**Proof.** Assume WLOG that \( i_1 \) constrains some instance symbol \( i' \), and \( i' \) is progressed before any other instance symbol constrained by \( i_1 \) or \( i_2 \) in the derivation \( N_j \rightarrow_{p_{IM}} N_k \), or that \( i_1 \) and \( i_2 \) constrain no tasks. Modify the derivation as follows: Let \( N_j \xrightarrow{i_1,i_2} N_j' \). Drop any decomposition or goal release of descendant symbols of \( i_2 \). For any operator application to a descendant of \( i_2 \), replace with task insertion. \( \Box \)

**Construction 5.3.** Let \( P \) be an HTN problem (by Construction 4.1 if necessary). Alford et al. [2015b] extends the work of Geier and Bercher [2011] to show that any solvable task-insertion problem can be solved with *acyclic progression*, which never decomposes a task’s ancestor of the same task name. We modify it for merging as follows: Where acyclic progression maintains a history of ancestral task names, instead associate each task with a counter, starting the counters in the initial node with \(|C|\) (the number of non-primitive task names). Decompose only tasks with a positive counter, and associate a decremented counter with the resulting children. When merging two tasks, the new task receives the greater of the two counters. Finally, after every decomposition or action application, all mergeable task names are merged.

So GTN_{15} planners can merge all mergeable tasks after every operation, leaving at most \(|C|\) unconstrained tasks, which is incompatible with HTN’s hardness proof. This leaves us with a looser bound for GTN_{15} planning:

**Theorem 5.4.** Deciding whether a problem is solvable under GTN_{15} semantics is \( \text{PSPACE-hard} \) and in \( \text{NEXPTIME} \).

Meanwhile, HGN_{1} planning is trivially in \( \text{PSPACE} \): with insertion, one only needs to guess a totally-ordered sequencing of the initial subgoals, and solve it as a series of sequential propositional planning problems [Bylander, 1994]:

**Theorem 5.5.** Deciding whether an HGN problem is solvable under GTN_{1} semantics is \( \text{PSPACE-complete} \).

6 Conclusion and Future Work

A number of hierarchical planning formalisms modify the standard HTN semantics in different ways; HGN planning uses goals in lieu of tasks, while ANML uses both tasks and goals as well as task insertion and sharing. Despite increased interest in using these planners in practice, there is little insight on how these modifications change the theoretical properties of HTN planning, both in terms of expressivity or computational complexity. Our key contributions against this are:

- We formalized GTN planning, a hybrid formalism that supports both task and goal decomposition. We also formalized task insertion and task-sharing semantics under this formalism, capturing features of HGNs and ANML.
- We provided plan-preserving translations from GTN problems with any combination of task insertion and sharing semantics to standard HTN semantics, thus showing that they are no more expressive than HTN planning, which in general is semi-decidable.
- We showed that HGN planning is also semi-decidable. In fact, we showed that HGN, HTN, and HTN_{15} planning are all equivalent complexity-wise.
- We provided plan-preserving translations from HGN problems to HTN problems with task sharing semantics, thus showing that HGN is no more expressive than HTN_{15}.
- Finally, we provided new upper bounds for subsets of GTN planning, showing that that HGN_{15} is in \( \text{PSPACE} \), and that GTN_{15} and GTN_{1} planning are in \( \text{NEXPTIME} \).

While we show that HGNs and GTNs (with and without task sharing and insertion) can be compiled to HTN planning, we do not claim that this is a superior method of planning. Goals, sharing, and insertion all provide explicit semantics that are obscured by the compilation process. This reduces both the ability to analyze the domain for correctness and the applicability of some heuristic search techniques [Shivashankar et al., 2016].

**Future Work.** There are several avenues to extend this work, some of which are:

- We showed only that GTN_{15} belong in \( \text{NEXPTIME} \). We conjecture GTN_{15} planning may be \( \text{PSPACE-complete} \).
- For HGN planning, we derived a semi-decidability result for partially ordered problems, which complements a known \( \text{EXPTIME-completeness} \) result [Shivashankar et al., 2012]. A more nuanced classification of HGN planning (similar to the classification of HTN planning [Alford et al., 2012]) would also extend to GTN planning.
- We provided plan preserving translations only in one direction, e.g. \( \text{HGN} \rightarrow \text{HTN}_{15} \), \( \text{GTN}_{15} \rightarrow \text{GTN} \), and \( \text{GTN} \rightarrow \text{HTN} \). We suspect some of the inverse plan-preserving translations are possible and others are not. Proving the non-existence of any of these translations would provide a firm separation in expressivity [Höller et al., 2016].
- Given the partially-ordered nature of GTN planning, it seems natural to extend it to planning with concurrent actions. Even PDDL 3 semantics [Gerevini and Long, 2005] for trajectory constraints assume linear planning, which does not appear to increase complexity; it is not clear if partial order planning plus trajectory constraints increases complexity.
References


