



# The negations of conjunctions, conditionals, and disjunctions



Sangeet Khemlani<sup>a,\*</sup>, Isabel Orenes<sup>b</sup>, P.N. Johnson-Laird<sup>c,d</sup>

<sup>a</sup> Naval Research Laboratory, United States

<sup>b</sup> Universidad de la Laguna, Tenerife, Spain

<sup>c</sup> Princeton University, United States

<sup>d</sup> New York University, United States

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## ABSTRACT

How do reasoners understand and formulate denials of compound assertions, such as conjunctions and disjunctions? A theory based on mental models postulates that individuals enumerate models of the various possibilities consistent with the assertions. It therefore predicts a novel interaction: in affirmations, conjunctions, *A and B*, which refer to one possibility, should be easier to understand than disjunctions, *A or B*, which refer to more than one possibility; in denials, conjunctions, *not(A and B)*, which refer to more than one possibility, should be harder to understand than disjunctions, *not(A or B)*, which do not. Conditionals are ambiguous and they should be of intermediate difficulty. Experiment 1 corroborated this trend with a task in which the participants selected which possibilities were consistent with assertions, such as: *Bob denied that he wore a yellow shirt and he wore blue pants on Tuesday*. Experiment 2 likewise showed that participants' own formulations of verbal denials yielded the same trend in which denials of conjunctions were harder than denials of conditionals, which in turn were harder than denials of disjunctions.

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## 1. Introduction

To deny an affirmative assertion is to negate it, and negation serves an important function in natural language (e.g., Horn, 2001) and in logic (e.g., Aristotle, 1984; Quine, 1974). Negation is also important in psychology since it is an abstract concept with a meaning outside any sensory modality (cf. Barsalou, 1999; Glenberg, Robertson, Jansen, & Glenberg, 1999; Hald, Hocking, Vernon, Marshall, & Garnham, 2013). Early psychological studies of negation focused on the interpretation of negative sentences in part because the then theory of transformational grammar introduced negation by way of a transformation (Klima, 1964). Their principal discovery, however, was semantic. Not only were negative assertions, such as, "The circle is not above the triangle", harder to verify than their affirmative counterparts, but there was an interaction between the polarity of an assertion (affirmative or negative) and its truth value (true or false): true affirmatives were easier to verify than false affirmatives, whereas true negatives were harder to verify than false negatives (Wason & Jones, 1963). This discovery led to the formulation of various information-processing theories of negation (e.g., Clark & Chase, 1972; Dale & Duran, 2011; Kaup, Zwaan, & Lüdtke, 2007; Orenes, Beltrán, &

Santamaría, 2014; Wason & Johnson-Laird, 1972). What studies have not considered, however, is the negation of different sorts of compound assertion, such as conjunctions ("and"), conditionals ("if\_ then\_"), and disjunctions ("or"). The present paper presents an investigation of them.

We carried out various preliminary studies, both online and face-to-face, which showed that naïve individuals – those who have not studied logic – have difficulty in understanding the task of "negating" assertions. For example, when we asked participants to list what was impossible given the "negation" of compound assertions, their performance was almost at chance. We therefore framed our experiments using a concept that they did understand: the denial of assertions. In linguistics, negation is a syntactic concept with semantic consequences. As Aristotle argued (see *De Interpretatione* in Aristotle, 1984, Vol. 1), negations contradict the negated assertion, i.e., they reverse its truth value: the negation of a true assertion is false, and the negation of a false assertion is true. But, negation can apply to constituents of sentences, and to interrogatives and imperatives. In contrast, denial is a speech act in which speakers correct assertions, not questions or requests, by negating affirmatives or unnegating negatives. In the context of our experiments, no difference exists between the following two sorts of instruction: please formulate a negation of this sentence, and please formulate a denial of this sentence, except that naïve individuals are much less likely to be confused by the latter instruction, because "negation" sounds like a syntactic command rather than a semantic one. Hence, in what follows, we will treat "denial" and "negation" as interchangeable.

\* Corresponding author at: Navy Center for Applied Research in Artificial Intelligence, Naval Research Laboratory, 4555 Overlook Ave. SW, Washington, DC 20375, United States. Tel.: +1 571 339 931.

E-mail address: [skhemlani@gmail.com](mailto:skhemlani@gmail.com) (S. Khemlani).

The paper begins with an account of negation from a logical standpoint, which we have based on Rips's (1994) psychological theory. Next, the paper describes a contrasting theory based on mental models. It then reports two experiments designed as crucial tests of the theories' predictions. Finally, it relates the results of the experiments to a general account of negation (Khemlani, Orenes, & Johnson-Laird, 2012).

### 1.1. The negation of compounds in logic

How do individuals understand the consequences of the negation, or denial, of compound assertions? If they know De Morgan's laws for interrelating the negations of conjunctions and disjunctions, they can apply the laws to infer a conclusion expressing the correct negation. These laws are embodied in Rips's (1994, p. 112 et seq.) PSYCOP theory, as follows:

1. NOT (P AND Q) implies (NOT P) OR (NOT Q)
2. NOT (P OR Q) implies (NOT P)
3. NOT (P OR Q) implies (NOT Q)

In these rules, OR is an inclusive disjunction, which allows that both disjuncts can be true. The rules can be used to work forwards from a premise to draw a conclusion. Rule (1) can also be used to work backwards from a given conclusion, but PSYCOP includes a single rule that combines (2) and (3) in order to work backwards to prove that a given conclusion, (NOT P) AND (NOT Q), follows from the premise (NOT P OR Q). To illustrate how De Morgan's rules work, suppose that you are asked for the consequences of the assertion:

4. It's not the case that Pat entered the room and she saw Viv.

Your first step is to grasp that its logical form is NOT (P AND Q), where P signifies *Pat entered the room* and Q signifies *Pat saw Viv*. Your second step is to find and to apply the corresponding formal rule of inference (1) to yield the conclusion: (NOT P) OR (NOT Q). And your final step is to restore the content as the values of the variables in the conclusion:

5. Pat didn't enter the room or she didn't see Viv.

PSYCOP predicts that it should be more difficult to determine the consequences of the negation of a disjunction, that is, to work forwards to a conclusion from:

6. It's not the case that Pat entered the room or she saw Viv.

You must use both rules (2) and (3), and the rule for forming a conjunction of their respective consequences. It follows that the denial of a conjunction should be easier to grasp than the denial of a disjunction. The following implication is valid in logic:

7. NOT (IF P THEN Q) implies P AND NOT-Q

Some proponents of formal rules of inference appear to accept such a rule. For example, Beth and Piaget (1966, p. 181) wrote that given a hypothesis of the form, *if p then q*, individuals should try to refute it by searching for a counterexample, *p and not-q*. But, rule (7) strikes many people, including Rips, as not intuitive, and so he excludes it from PSYCOP. It follows, as Rips proves, that PSYCOP cannot make the following sort of inference:

8. It's not the case that if Pat entered the room then she saw Viv.

So, Pat entered the room and she didn't see Viv. Such inferences could be proved only if such rules as (7) are added to the system (Rips, 1994, p. 128). Presented with the inference in (8), PSYCOP itself halts but without a proof that the conclusion follows from the premise. In summary, formal rules of inference lead to the psychological prediction that the denial of a disjunction should be harder than the denial of a conjunction, and the denial of a conditional should be hardest of all, if not impossible.

### 1.2. Mental models and the negation of compounds

The theory of mental models – the “model” theory for short – differs in several ways from an account based on formal rules of inference. The model theory neither extracts logical forms nor applies formal rules of inference to them. Instead, the model theory postulates that individuals grasp the significance of an assertion when they know the *possibilities* to which it refers (Johnson-Laird, 1983; Johnson-Laird & Byrne, 1991). The mind constructs mental models of these possibilities. We now explore how the theory treats various compound assertions and their denials.

A conjunction, such as (9a), refers to a single possibility in which both clauses hold, whereas its denial (9b) refers to three possibilities:

- 9a. Pat entered the room and she saw Viv.
- 9b. It's not the case that both Pat entered the room and she saw Viv.

We list the three possibilities for (9b) on separate rows and abbreviate them as follows:

–P	–V
–P	V
P	–V

where ‘–’ denotes negation, *P* stands for *Pat entered the room*, and *V* stands for *Pat saw Viv*. (We use letters in these diagrams for convenience; in reality, people build models of the world.) In contrast, consider a disjunction and its negation:

- 10a. Pat entered the room or she saw Viv.
- 10b. It's not the case that Pat entered the room or she saw Viv.

Given an inclusive interpretation, the disjunction (10a) refers to three possibilities:

P	–V
–P	V
P	V

Its negation (10b) refers to only one possibility:

–P	–V
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Given an exclusive interpretation, however, both the affirmation of the disjunction and its denial refer to two possibilities: *P and V* shifts from an affirmative possibility to a negative one. As the preceding examples illustrate, the negation of the models of an affirmative assertion yield the models of the corresponding negative assertion, where the negation of the models are their complement in the set of all possible models based on the relevant atomic propositions.

Conditional assertions are more complicated and more controversial than the preceding compounds (see, e.g., Evans, 2007; Handley, Evans, & Thompson, 2006; Johnson-Laird, Byrne, & Girotto, 2009). Unlike conjunctions and disjunctions, conditional assertions, such as (11), contain a subordinate clause (the *if*-clause) and a main clause (the *then*-clause),

11. If she entered the room then Pat saw Viv.

One sign of a subordinate clause is that, as in this example, a pronoun can refer forwards to the same referent as a noun phrase in the subsequent main clause. Such a “cataphorical” reference, however, is not

possible from one main clause to another, and so “she” doesn’t refer to the same individual as “Pat” in this example:

12. She entered the room and Pat saw Viv.

The model theory postulates that for a conditional, such as (11), individuals normally represent one possibility explicitly and the rest in a single implicit model:

P	V
...	

where *P* denotes Pat entering the room and *V* denotes her seeing Viv. In tasks such as enumerating possibilities (see Table A2 in Barres & Johnson-Laird, 2003), some individuals list only the single possibility corresponding to the explicit mental model above, others list the two possibilities corresponding to a biconditional interpretation (“if, and only if”):

P	V
–P	–V

and still others list the three possibilities corresponding to a conditional interpretation:

P	V
–P	V
–P	–V

These three interpretations correspond to a developmental trend in children’s interpretations (see, e.g., Barrouillet, Grosset, & Lecas, 2000). An additional layer of complexity, which we avoided in the present experiments, is that the meaning of the clauses, their referents, and general knowledge, can modulate the interpretation of conditionals in many ways (Byrne & Johnson-Laird, 2009; Johnson-Laird & Byrne, 2002), and so the process of their interpretation cannot be truth functional (pace Handley et al., 2006).

Negation applied to assertions containing a subordinate and a main clause is often interpreted as concerning only the main clause (Byrne & Johnson-Laird, 2009; Johnson-Laird et al., 2009; Khemlani et al., 2012). It has an interpretation in which an assertion such as:

13. It’s not the case that if she entered the room then Pat saw Viv.

is taken to mean:

14. If she entered the room then Pat did not see Viv.

Some authors allow for this interpretation with a negation in the main clause, but argue that nevertheless it signifies a large scope interpretation (e.g., Politzer, Over, & Baratgin, 2010). But, in these cases in which negation occurs the main clause can receive the small scope interpretation. Indeed, this interpretation applies to many sentential operators. It allows assertions such as, *Slowly, if he started the race quickly, he ended it*, to have a sensible interpretation in which he ended the race slowly.

The small scope interpretation of negation may be easier to understand than the negation of the full assertion, but it is incorrect. As Aristotle argued (see *De Interpretatione* in Aristotle, 1984, Vol. 1), negations contradict the negated assertion, i.e., they reverse its truth value. But, small scope negations violate this criterion for conditionals, as the following examples illustrate:

15. If they are Democrats then they are honest.

If they are Democrats then they are not honest.

Given that “they” refers to the same set of individuals in both assertions, the two conditionals cannot both be true, but they can both be false – as

they would be when some of the relevant Democrats are honest and some of them are not. The two assertions make contrary assertions about the Democrats, not contradictory assertions. Nevertheless, reasoners often appear to make the small scope negation (Handley et al., 2006).

The correct negation of a conditional assertion refers to any possibility that is incompatible with the conditional. In example (13), only one such possibility exists, i.e., the possibility in which Pat entered the room but she did not see Viv. As we saw earlier, some formal theorists take this inference to be the heart of hypothesis testing (Beth & Piaget, 1966, p. 181), whereas other formal theorists take it to verge on the paradoxical (Rips, 1994, p. 125). The facts are that some individuals make the correct large-scope interpretation, whereas others make the small-scope one (Giroto & Johnson-Laird, 2004; Johnson-Laird et al., 2009).

Consider a negated conjunction, such as:

16. It’s not the case that Pat entered the room and she saw Viv.

which we abbreviate as: *Not (P and V)*. Individuals do not immediately know the possibilities to which such negated compounds refer, and so they have to infer them. The correct inference depends on formulating the negation of the models of the corresponding affirmative compound, i.e., their complement in the set of all possible models based on the same atomic propositions. Hence, *Not (P and V)* has the three models other than *P and V* in the set. The model theory postulates that they construct these models, or attempt to do so, using a process of “enumerative negation” in which they make a series of independent negations of the clauses in compounds. They begin with the possibility in which the negation is applied to each clause of the negated conjunction in (16): *not-P and not-V*. This possibility is not consistent with the original affirmative conjunction, *P and V*, and so they realize that it is one possibility in which the negation holds. At this point, some reasoners may stop, with the result that they consider only this initial possibility. But, if they continue, they apply the negation to only one of the clauses, e.g., *not-P and V*. They can detect that it too is inconsistent with the original affirmative and accordingly a possibility consistent with the negation. Likewise, they may grasp that *P and not-V* is also a possibility that renders the negation true. Finally, reasoners need to consider the case, *P and V*. The possibility is consistent with the unnegated conjunction, and it is therefore inconsistent with the negation of the conjunction. The same general procedure applies to all sentential connectives between main clauses, and in human it can be applied recursively to clauses within clauses, though human reasoners have difficulty in such cases (Barres & Johnson-Laird, 2003).

An enumerative negation of a disjunction yields only a single case: *not-P and not-V*, because all the other putative negations are consistent with the original affirmative disjunction. The model theory postulates that intuitions are based on single mental models (Johnson-Laird, 1983, Ch. 6), and that individuals prefer to reason on their basis (Johnson-Laird & Byrne, 1991, p. 60). It is striking that the only negative connective between main clauses in English refers to the single possibility in the negation of a disjunction: *neither P nor V*. The correct negations of other compounds call for mental deliberation.

An enumerative negation of a conditional depends on its interpretation. We have already described the contrast between a large scope interpretation: *Not (if P then V)*, and a small-scope interpretation: *If P then not V*. But, as we have also shown, there is another ambiguity between interpreting “if” as a conditional, which refers to three possibilities, or as a biconditional, “if and only if”, which refers to only two possibilities. Granted that more possibilities entail more work (see, e.g., Johnson-Laird, 1983), a bias for interpretations of affirmative conditionals as biconditionals is likely. But, when a conditional is negated, a negative conditional yields only one possibility: *P and not-V*, whereas a negative biconditional yields two possibilities, and so the bias should favor a conditional interpretation. What is most unlikely is that individuals will enumerate all three possibilities to which *If P then not-V* refers.

The model theory makes a novel, and perhaps counterintuitive, prediction about compounds. Because more models entail more work, individuals tend to make interpretations that minimize models – even

in a simple task, such as enumerating the possibilities to which assertions refer. For affirmative assertions, conjunctions (one possibility) should be easier to understand than conditionals (one possibility for the single explicit mental model, two possibilities for biconditional interpretations, and three possibilities for conditional interpretations), which in turn should be easier to understand than disjunctions (two models for an exclusive interpretation and three possibilities for an inclusive interpretation). This order, however, reverses for the negations of compounds. For negative assertions, disjunctions (one possibility) should be easier to understand than conditionals (one possibility for large scope interpretations, and three possibilities for small scope interpretations), which in turn should be easier to understand than conjunctions (three possibilities).

The prediction presupposes that the greater the number of models of possibilities to which an assertion refers, the harder it should be to understand the assertion. Hence, it hinges on the theory that individuals construct mental models of possibilities, and that the core meaning of negation refers to the set of possibilities complementary to those to which the corresponding affirmative assertion refers (Khemlani et al., 2012). In contrast, as we showed earlier, a theory based on formal rules of logic (Rips, 1994) predicts an increasing trend in difficulty from conjunctions through disjunctions to conditionals, whether they are affirmative or negative. We now describe two experiments designed to test the contrasting predictions.

**2. Experiment 1: the comprehension of sentential negations**

Experiment 1 tested the predictions of the model theory using a task in which the participants listed what is possible given affirmations and denials of three sorts of compound: *A and B*, *A or B*, and *if A then B*. As we explained earlier, we framed the negation task as one concerning the denial of assertions, because preliminary studies showed that naïve individuals found the term, “negation” confusing. The participants accordingly judged whichever of four cases: *A and B*, *A and not-B*, *not-A and B*, *not-A and not-B*, was “possible” given an assertion, which was either an affirmation or a denial of one of the three sorts of compound.

**2.1. Method**

**2.1.1. Participants**

22 adult native English-speaking individuals were recruited on an online system, Mechanical Turk, hosted by Amazon.com that allows people to participate in on-line experiments for monetary compensation (see Paolacci, Chandler, & Ipeirotis, 2010, for an evaluation of this experimental platform).

**2.1.2. Design, materials, and procedure**

The participants acted as their own controls and selected what was possible for three affirmative assertions (based on *and*, *or*, and *if*) and their three denials, i.e., they carried out six problems in total. The assertions were presented as a block of affirmations and a block of denials in a counterbalanced order in two groups. The actual assertions concerned the color of the clothes of an individual, Bob, who affirmed or denied what he wore on a particular day, e.g.:

- 17. Bob [asserted/denied] that he wore a yellow shirt [and/or] he wore blue pants on Monday.
- 18. Bob [asserted/denied] that if he wore a red shirt then he wore pink pants on Monday.

We used adverbial phrases, such as “on Monday”, to convey that the assertion was about what the person wore on a particular occasion. We did not try to disambiguate whether disjunctions were inclusive (“or both”) or exclusive (“but not both”), because the theory predicts an exclusive interpretation for affirmatives but an inclusive interpretation for denials, which minimize the number of possibilities in both cases. For each of the preceding examples, the participants selected whichever of the following cases they judged to be possible given the assertion:

- 19. Bob wore a yellow shirt and he wore blue pants.  
Bob wore a yellow shirt and he wore non-blue pants.  
Bob wore a non-yellow shirt and he wore blue pants.  
Bob wore a non-yellow shirt and he wore non-blue pants.

The participants were told to select all the cases that they judged to be possible for each assertion. The order of presentation of the four cases was counterbalanced over the trials.

**2.2. Results and discussion**

No reliable difference occurred in the accuracy of the responses in the two groups counterbalancing the order of the blocks, and so we pooled the data for analysis. Table 1 shows the percentage of participants' most frequent judgments of possibility depending on the compound and the polarity of the task. Overall, 74% of responses corresponded to the model theory's predictions, and 20 out of the 22 participants made more than 50% of predicted responses (binomial test of 20 out of 22 with a conservative prior of .5,  $p < .0001$ ).

For affirmations, conjunctions yielded 86% correct interpretations, disjunctions yielded 59% exclusive interpretations and 9% inclusive interpretations, and conditionals yielded 45% conditional interpretations and 18% biconditional interpretations. (Conditionals also yielded 27% one model interpretations, equivalent to conjunctions, as observed in, e.g., Barres & Johnson-Laird, 2003.) As the table shows, 41% of the

**Table 1**  
The percentages of the different denials of conjunctions, conditionals, and disjunctions in Experiment 1.

Connective	Schematic	Contingencies rated by participants as “possible”							Misc.	
		<i>A B</i>	<i>A B</i>	<i>A B</i>	<i>A -B</i>	<i>A -B</i>	<i>-A B</i>	<i>-A B</i>		
			<i>-A B</i>		<i>-A B</i>		<i>-A -B</i>			
			<i>-A -B</i>	<i>-A -B</i>		<i>-A -B</i>	<i>-A -B</i>			
<b>Affirmatives (Bob affirmed that ...)</b>										
Conjunction	... he wore <i>A and B</i>	<b>86</b>								14
Conditional	... if he wore <i>A then he wore B</i>	27	<b>45</b>	<b>18*</b>						10
Disjunction	... he wore <i>A or B</i>				<b>59</b>					41
<b>Denials (Bob denied that ...)</b>										
Conjunction	... he wore <i>A and B</i>				45	<b>18</b>	18	14		5
Conditional	... if he wore <i>A then he wore B</i>					<b>59*</b>			<b>14</b>	27
Disjunction	... he wore <i>A or B</i>				<b>86</b>					14

Note: The asterisks indicate two cases that were labeled as correct given a liberal interpretation of participants' responses. For affirmative conjunctions, the 18% of participants who inferred that “*A and B*” and “*not-A and not-B*” were possible made a biconditional interpretation of the conditional. Likewise, for denials of conditionals, the 59% of participants who inferred that “*A and B*”, “*not-A and B*”, and “*not-A and not-B*” were all possible made a “small scope” interpretation (consistent with Khemlani et al., 2012). The last column presents percentages of miscellaneous errors, i.e., the pooled percentage of different errors that occurred on less than 10% of trials. Numbers in bold indicate logically correct responses.

disjunctions, *A or B*, yielded various miscellaneous responses, which each occurred on less than 10% of trials. They included the 9% of inclusive disjunctions mentioned above, 18% of cases in which only a single conjunction was selected as possible, either *A and B* or *A and not-B*, and 14% of errors in which *not-A and not-B* was selected as possible. Affirmations of a disjunction are plainly quite tricky to grasp.

The pattern of results was quite different for denials. Conjunctions yielded only 18% correct responses, conditionals yielded small-scope interpretations, *if A then not-B* (59%) and large scope interpretations, *A and not-B* (14%), and disjunctions yielded 86% correct inclusive responses. In sum, the predicted interaction between polarity and the connectives was reliable for the percentages of correct responses (Wilcoxon test,  $z = 1.83$ ,  $p = .03$ , Cliff's  $\delta = .25$ ). The interaction was also significant for the compounds containing only main clauses, i.e., for conjunctions and disjunctions (Wilcoxon test,  $z = 3.59$ ,  $p = .0005$ , Cliff's  $\delta = .68$ ).

An obvious characteristic of performance was the participants' bias to minimize the number of possibilities. The compound, *A or B*, was treated as exclusive (two models) rather than inclusive (three models) in affirmations, but as inclusive (one model) rather than exclusive (two models) in negations. The compound, *If A then B*, was often treated as a biconditional (two models) in affirmations, but never in negations. Likewise, in denials of conjunctions, the participants mainly judged *not-A and not-B* alone as possible (45%), and 14 out of the 22 participants thought of only one possibility (binomial  $p < .005$ , given a prior probability of .33).

### 3. Experiment 2: the formulation of sentential negations

The previous experiment investigated the comprehension of affirmations and denials of compound assertions, by examining the possibilities to which individuals took them to refer. In contrast, the present experiment investigated individuals' own verbal formulations of denials of compound assertions. A preliminary study showed that when individuals are asked to "negate" a conditional, they tended to negate both of its clauses (69% of trials; see also Espino & Byrne, 2012). Hence, the present task was instead to "deny" an assertion. The participants had to formulate denials of three sorts of assertion: *A and B*, *A or B*, and *If A then B*. The model theory predicts that they should construct a set of models of possibilities, each of which is conjunctive in form, and retain those that are inconsistent with the original assertion. It follows that the participants should tend to be most accurate in denying disjunctions, because the first conjunction that they are likely to formulate, *not-A and not-B*, is the one and only correct denial (of an inclusive disjunction). They should be less accurate with conditionals, because they are likely to have to construct more than one conjunction before they encounter the correct denial: *A and not-B*. And they may take the negation to apply only to the then-clause, and accordingly express it as: *if A then not-B*. Finally, the participants should tend to be least accurate with conjunctions, because their correct denial depends on enumerating three models of possibilities: *not-A and not-B*, *not-A and B*, and *A and not-B*. Those familiar with logic, however, could use De Morgan's law to respond: *not-A or not-B* (see Rips, 1994).

### 3.1. Method

#### 3.1.1. Participants

A new sample of 21 native English-speaking participants was tested from the same population as before.

#### 3.1.2. Design, materials, and procedure

The participants acted as their own controls and had to formulate denials of six conjunctions, six disjunctions, and six conditionals, which were presented to each of them in a different random order. A typical assertion to be denied was:

20. Mary likes espresso and she enjoys biscotti.

Each clause in the assertions to be denied contained a transitive verb, and two noun phrases, with at least one based on a proper noun. One noun phrase in the second clause was co-referential with a noun phrase in the first clause. The resulting compounds were accordingly simple everyday assertions. No proper noun, noun phrase, or transitive verb, occurred more than once in the materials. Participants were instructed to deny the assertions by formulating a complete assertion that began with the word, *No*, as a preface to their denial, and the assertion could be of any length. Denials of disjunctions (*A or B*) were counted as accurate if they captured the conjunctive relation, *not-A and not-B*. Denials of conditionals (*if A then B*) were counted as accurate if they captured the conjunctive relation, *A and not-B*. And denials of conjunctions (*A and B*) were counted as accurate if they captured the disjunctive relation, *not-A or not-B*.

### 3.2. Results and discussion

Table 2 presents the percentages of the various sorts of denial. The participants corroborated the predicted trend: they made correct denials for 67% of disjunctions (*not-A and not-B*), for 28% of conditionals (*A and not-B*), and for 0% of conjunctions (*not-A or not-B*). The predicted trend was highly reliable (Page's  $L = 281.5$ ,  $z = 4.55$ ,  $p < .00001$ ). As in Experiment 1, most participants formulated denials of conjunctions by listing only a single possibility, *not-A and not-B* (66% of responses). This again suggests a propensity to minimize the number of models.

The conditionals elicited 34% of denials of the form: *If A then not-B*, which is consistent with the hypothesis that reasoners reduce the scope of the negation to make it easier to comprehend (Khemlani et al., 2012). The participants making this response tended to differ from those who made the correct denials: 7 out of the 21 participants responded *if A then not B* on half or more of the trials, and 10 out of the 21 participants responded *A and not B* on half or more of the trials. The difference between these two post-hoc groups in the frequency with which they responded *if A then not B* was highly reliable (Mann-Whitney test,  $z = 3.50$ ,  $p < .0001$ , Cliff's  $\delta = 1.0$ ). On 21% of trials, participants denied a conditional merely by asserting, *not B* – a response which also suggests that they presupposed the if-clause and denied the then-clause.

As the theory predicts, when the participants had to deny assertions, they were most accurate in denying disjunctions and least accurate in

**Table 2**

The percentages of the different denials of disjunctions, conditionals, and conjunctions in Experiment 2.

Assertion to be denied	Denials formulated by participants					
	<i>No, not A and not B</i>	<i>No, A and not B</i>	<i>No, if A then not B</i>	<i>No, not A</i>	<i>No, not B</i>	Misc.
Disjunction: <i>A or B</i>	<b>67</b>	2	0	11	2	18
Conditional: <i>If A then B</i>	9	<b>28</b>	34	3	21	5
Conjunction: <i>A and B</i>	66	9	0	8	6	11

The last column presents percentages of miscellaneous errors, i.e., the pooled percentage of different errors that occurred on less than 10% of trials. Numbers in bold indicate logically correct responses.

denying conjunctions. Skeptics might argue that a perfectly good denial of a conjunction, such as (21) above, is:

Mary doesn't like espresso.

Indeed, it would be an acceptable denial if an individual knew for a fact that Mary didn't like espresso. However, our participants had no such knowledge, and so, in its absence, a response based solely on one possibility and ignoring the other possibilities is erroneous. Moreover, even if we counted: *No, not A* and *No, not B* as correct denials of *A and B*, the conjunctions remain harder to deny than disjunctions (67% vs. 14%).

#### 4. General discussion

The affirmation of a conjunction is easier to understand than the affirmation of a disjunction, and the same effect occurs in inferences (García-Madruga, Moreno, Carriedo, Gutiérrez, & Johnson-Laird, 2001). The affirmation of a conditional is also quite difficult to grasp explicitly, because individuals normally do not represent cases in which the if-clause is false. When they need to do so, as in listing the possibilities to which assertions refer, they often treat a conditional as a biconditional. In contrast, the denial of a conjunction is harder to understand than the denial of a conditional, which in turn is harder to understand than the denial of an inclusive disjunction (Experiment 1). The assertions in our experiments introduce no temporal or causal relations, or any effects in which the meanings or referents of clauses modify the interpretation of sentential connectives (see Johnson-Laird & Byrne, 2002). The theory of mental models accordingly predicts this novel interaction on the grounds that individuals construct models of the possibilities to which assertions refer, and that they are biased to minimize the number of models. A recent study by Macbeth and colleagues further corroborated the prediction using tasks in which participants selected an appropriate compound assertion that was equivalent to a negated conjunction or to a negated disjunction (Macbeth, Razumiejczyk, Crivello, Fioramonti, & Girardi, 2013; Macbeth et al., 2014).

Individuals do not know the possibilities to which all but the simplest negated compounds refer, and so they have to infer them. In principle, they have to construct the complement of the models of the corresponding affirmative compound: their complement in the set of all possible models based on the same atomic propositions. The model theory postulates that for denials they do so using a process of enumerative negation in which they construct a sequence of conjunctive models of negated possibilities, checking that they render the original affirmative assertion false (see Khemlani et al., 2012). The model theory accordingly predicts that a conjunction, *A and B*, has one model of a possibility: *A B*; whereas its denial, *not (A and B)*, calls for the negation of this model, which yields three models:  $\neg A \neg B$ ,  $\neg A B$ ,  $A \neg B$ . However, the model theory postulates that intuitions are based on a single model (see Section 1.2), and inferences based on disjunctions often depend on just a single model (Johnson-Laird & Byrne, 1991, p. 60; Barres & Johnson-Laird, 2003). Hence, the process of constructing the models for the denial of a disjunction is biased to stop after constructing *not-A and not-B*, as occurred on 45% of trials in Experiment 1 and 66% of trials in Experiment 2. This tendency to negate conjunctions by negating each conjunct in the conjunction could be a heuristic, as a reviewer suggested. Certainly, it is an intuitive strategy, but, as the model theory predicts, some individuals do construct more than one possibility. A conditional, *If A then B*, has two or three models of possibilities depending on whether it receives a biconditional or conditional interpretation, and sometimes elicits only a single one corresponding to its one explicit mental model. Its denial does not readily yield the correct negation, *A and not-B*, because *if* occurs in a subordinate clause (see Quine, 1974, p. 19), and as a result the denial may be assigned solely to the then-clause: *if A then not-B* (Byrne & Johnson-Laird, 2009; Khemlani et al., 2012). A disjunction, *A or B*, is more likely to elicit an exclusive interpretation of two models than an inclusive interpretation of three models,

whereas its denial switches in favor of a denial of an inclusive interpretation yielding just one model.

Some psychologists have argued that *if A then not B* is the correct denial of *if A then B* (Handley et al., 2006). But, this defense has a drawback: it no longer treats a negation as contradicting the corresponding affirmative. It also offers no principled explanation of why some individuals do take *A and not-B* to be the denial of a conditional, or why most people take this case to falsify a conditional too (Espino & Byrne, 2012; Evans, Newstead, & Byrne, 1993; Johnson-Laird & Tridgell, 1972). Hence, the model theory treats the correct negation of the conditional as *A and not-B* (Khemlani et al., 2012), but allows the small-scope interpretation equivalent to *If A then not-B*. Despite the occurrence of negation in the main clause, readers might suppose that it still has a large scope (cf. Politzer et al., 2010), but, as we showed in the introduction, it is common for sentential operators, such as negation, to have a small scope interpretation when they occur in main clauses of compounds containing subordinate clauses. For example, negation in the following assertion has a small scope:

21. After Pat entered the room, she didn't see Viv.

That is, the assertion presupposes that Pat entered the room. Likewise, as our results show, the negation in the following conditional is readily interpretable as having a small scope:

22. If Pat entered the room, she didn't see Viv.

Logicians are familiar with De Morgan's rules for the formulation of denials: *not (A and B)* is equivalent to *not-A or not-B*; and *not (A or B)* is equivalent to *not-A and not-B*. When naïve individuals have to formulate their own denials, they do not know these rules, and so they have to infer the denials of assertions. As the theory predicts, denial is harder for conjunctions than for disjunctions, and of intermediate difficulty for conditionals (Experiment 2). This result is contrary to the PSYCOP theory (Rips, 1994, p. 113), which predicts that denials of conjunctions should be easier than denials of disjunctions based on its formal rules for De Morgan's laws. For rules that work forwards from premise to conclusion, a single step yields the inference from the negation of a conjunction, whereas three steps based on different rules are needed for the inference from the negation of a disjunction. Likewise, the PSYCOP theory predicts that denials of conditionals should be the most difficult to interpret and formulate. Neither of these patterns was borne out in the data.

Defenders of formal rules might argue that the task of identifying possibilities (in Experiment 1) is biased towards the model theory, because possibilities themselves are central to the theory whereas they are peripheral to the formal rule theory. However, both theories do make predictions about the task, and so the task serves as a neutral arbiter. Even if the formal rule theory did not predict performance in the task, a comprehensive theory of reasoning should make predictions about such a task. Finally, there is nothing intrinsic to the task itself that would yield the predicted interaction, and so the task that we adopted suffices as a suitable test of the model theory.

In conclusion, the model theory may be unique in its prediction of an interaction between polarity and compound assertions. Affirmations are easier to understand, and to formulate, for conjunctions than for disjunctions. In contrast, negations are easier to understand, and to formulate for disjunctions than for conjunctions. In both cases, conditionals are of intermediate difficulty.

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