Detection and Communication with Entanglement

Tanner Crowder
Center for High Assurance Computing
The Naval Research Laboratory
Washington D.C, USA
tanner.crowder@nrl.navy.mil

Marco Lanzagorta
Center for High Assurance Computing
The Naval Research Laboratory
Washington D.C, USA
marco.lanzagorta@nrl.navy.mil

***Abstract*—We provide a numerical model for the probability of detecting a test object using entangled states in the optical regime. In this model, we will consider objects with varying optical properties. Using these results, we propose a new communication protocol based on quantum illumination.**

***Index Terms*—Quantum illumination, Quantum Communication, Entanglement.**

I. Introduction

The engineering challenges of building quantum enhanced technologies have proven great. The speed up over classical computation and the enhanced resolution of a quantum sensor, to name two, have not been fully realized due to the complexity of maintaining, manipulating, and measuring quantum states. However, in 2016, China reported an experimental implementation of a quantum radar that was able to detect stealth targets [1], [2]. While a staggering result, it is difficult to verify as the details were not made public. We will not foray into the debate about the applications or usefulness of a quantum sensor, but instead, using specifications similar those of the state-of-the-art, commercially available technologies and the standard theory of quantum illumination [3], we will numerically model the probability of detecting a circular test object in the optical regime. We will discuss how varying the reflectivity and roughness of an object could affect it's detectability, as well as the challenges of working in the optical regime; as we will show in Section II, the individual contributions of the object's transmissivity and absorptivity are inconsequential to our model and will not be considered. Lastly, in Section V, we propose an inconspicuous communication protocol that requires little-to-no power from the sender and uses similar technology.

To model the probability of detecting an object, we will assume that we are probing the environment with entangled pairs and are measuring the reflected states with single photon detectors. Then, the three main components we will consider are the entanglement source, the measurement apparatus, and the roughness and reflectivity of the test object. In our model, we will assume the source of entanglement is given by spontaneous parametric down conversion (SPDC). Currently, the detectors with the highest quantum efficiency are superconducting nanowires. Superconducting nanowires have a high quantum efficiency and low dark counts, with the caveat that they must be cryogenically cooled. They are also larger, more expensive, and more delicate than other single photon detectors, like single photon avalanche diodes, which can operate efficiently at room temperature. Superconducting nanowires have efficiencies ranging from 65% to 80%, and dark counts on the order of 1Hz-2kHz [4]–[6]. For our analysis, we will assume a single photon detector which has 80% quantum efficiency and a dark count rate of 1Hz. These specifications are similar to those that IdQuantique reports for their customizable superconducting nanowires [4]. In our analysis, we will use a standard noise model [7], [8] and will model the probability of detecting a circular test object of various sizes and reflectivity.

There are many challenges to using quantum states to detect the presence of an object; one of which is temporally separating the states. An ideal source of entanglement would produce entangled pairs on demand; that is, with near 100% certainty the source produces exactly 1 entangled pair at the push of a button; however, such sources are in their infancy and not commercially available. Using a pulsed laser through a non-linear crystal, one can generate entangled pairs via SPDC. Pulsing the laser allows one to timestamp the event; however, each pulse has a very low probability of containing a photon pair [9]. Even with this limitation, using a PPKTP crystal and a 2 ps pulsed laser, [5] and [6] generated 10^5 entangled pairs per second for each milliwatt of pump power. For our model, we will assume a pump power of 100mW, and we will therefore assume the capability of generating 10^7 entangled pairs per second.

II. Energy Flux Equations

We will build our model from the strength of the received signal from our test object, which is determined through an energetic analysis that compares the transmitted and received signal energies. We will assume that the transmitter is a highly collimated laser with a beam divergence θ and that our target has cross section σ. Usually, the beam divergence is small, so we can approximate the illuminated area as

$$A_i \approx \pi \left(\frac{w + R\theta}{2} \right)^2, \tag{1}$$

where R is the range and w is beam waist. As long as the illuminated area is smaller than the cross section of the target ($A_i < \sigma$), we can write the energy equation as

$$P_r = P_t \times e^{-2\chi R} \times \mathcal{R} \times \frac{A_r}{4\pi R^2}, \tag{2}$$

where P_t is the emitted power, the second term is the atmospheric attenuation to and from the object at a range R, the third term is the isotropic radiation scattered by a target of reflectivity $\mathcal{R}$, and finally, the fourth term is the portion of the isotropic power received by an optical element of area A_r. Eqn. 2 is valid as long as

$$R < \frac{2\sqrt{\frac{\sigma}{\pi}} - w}{\theta}. \tag{3}$$

A surface is consider rough/diffuse if the size of the surface irregularities are comparable or greater than the wavelength of the illuminating signal. If the object is diffuse, we can just consider the radiation diffused on the half plane towards the detector:

$$P_{rd} = P_t \times \mathcal{R}\ e^{-2\chi R} \times \frac{A_r}{4\pi R^2} \times 2\pi = \frac{P_t A_r \mathcal{R}\ e^{-2\chi R}}{2R^2}. \tag{4}$$

However, if the object is a smooth surface, and the scattering is mostly in the specular direction, then we can write

$$P_{rs} = P_t \times \mathcal{R}\ e^{-2\chi R}, \tag{5}$$

as long as the area of the detector A_r is larger than the illuminated area at a distance $2R$:

$$A_r > \pi \left(\frac{w + 2R\theta}{2}\right)^2 \Longrightarrow R < \frac{2\sqrt{\frac{A_r}{\pi}} - w}{2\theta}. \tag{6}$$

In the above expressions, the cross sections for diffuse (d) and smooth (s) flat targets of area A are given by

$$\sigma_d = 4A \text{ and } \sigma_s = \frac{4\pi A^2}{\lambda^2}. \tag{7}$$

III. Signal to Noise Ratio

The total power transmitted by the sensor is

$$P_t = Nh\nu, \tag{8}$$

where N is the number of photons transmitted per unit time, h is Planck's constant, and ν the frequency. Using results similar to those in [5], [6], [9], we will assume we can generate $N = 10^7$ entangled pairs per second.

The principal sources of noise are the shot noise N_{sn}, the solar background radiation N_{br}, and the dark counts N_{dc}. So we can write the total number of noise photons N_t as

$$N_t = N_{sn} + N_{br} + N_{dc}. \tag{9}$$

Then, the total power due to noise is given by $P_N = h\nu N_t$. We can approximate these expressions as

$$\begin{aligned} N_{sn} &\approx 1 \\ N_{br} &\approx \eta N_b \\ N_{dc} &\approx \frac{\sqrt{2R_{dc}}}{\eta} \end{aligned} \tag{10}$$

where η is the quantum efficiency of the detector, and R_{dc} is the dark count rate of the detector. The number of noise ("thermal") photons due to the solar background radiation is given by Planck's formula

$$N_b = \frac{1}{e^{h\nu/kT} - 1} \tag{11}$$

where k is Boltzmann's constant and T is the sun's temperature ($T \approx 6,000°K$).

Bringing all the equations together, we can write the signal-to-noise ratio as

$$\mathcal{S} = \frac{\eta P_r}{P_N}, \tag{12}$$

and there are two cases to consider.

First, if the object is diffuse and

$$R^2 < 4\sigma_d/\pi\theta^2 = 16A/\pi\theta^2, \tag{13}$$

then

$$\mathcal{S}_d = \frac{N\eta\mathcal{R}A_r e^{-2\chi R}}{2R^2(1 + \eta N_b + \frac{\sqrt{2R_{dc}}}{\eta})} \tag{14}$$

On the other hand, if the object is smooth and

$$R < \min\left\{\frac{2\sqrt{\frac{\sigma}{\pi}} - w}{\theta}, \frac{2\sqrt{\frac{A_r}{\pi}} - w}{2\theta}\right\}, \tag{15}$$

then

$$\mathcal{S}_s = \frac{N\eta\mathcal{R}e^{-2\chi R}}{1 + \eta N_b + \frac{\sqrt{2R_{dc}}}{\eta}} \tag{16}$$

Using the standard quantum estimation and detection techniques [3], [10], [11], we obtain the detection probability equations in terms of the signal-to-noise ratio $\mathcal{S}$:

$$\begin{aligned} P_{qo} &= 1 - e^{-\mathcal{S}} \\ P_{qs} &= 1 - e^{-\mathcal{S}/2} \\ P_c &= 1 - e^{-\mathcal{S}/4} \end{aligned} \tag{17}$$

for entangled (q) and non-entangled (c) signals. Note that P_{qo} is based on an optimal receiver described in [3], and has not been experimentally realized; however, there is a promising procedure outined in [12] which saturates the quantum Chernoff bound. For a more conservative estimate, we also model P_{qs} based on a suboptimal receiver [11].

IV. Discussion

Using Eqns. 14-17, we will provide a numerical analysis of how varying the reflectivity and roughness of the object could affect the probability of detection. For comparison purposes, we will consider both an entangled source and a non-entangled single photon source. Since we are modeling an optical signal, the surface irregularities of an object can be significant, and the diffuse scattering will most likely dominate [13]; in which case, when computing the probability of detecting most objects, one would use the diffuse terms from Sections II and III. To illustrate the effect diffuse scattering can have on the range, we will model objects that scatter both diffusely and specularly. We will assume the capability of producing 10 MHz of entangled pairs at 800 nm. We will assume the diameter of the beam is $1/4$ m, the divergence angle is

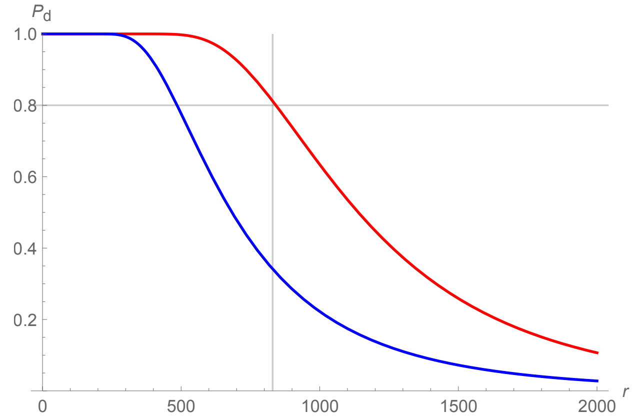

Fig. 1. Probability of detection plotted against the object's distance in meters for a diffuse object of reflectivity 1/2. The red line models optimal detection using entangled light, while the blue line models non-entangled light.

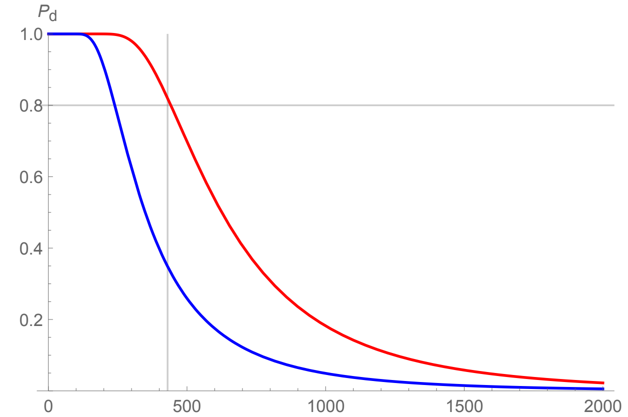

Fig. 2. Probability of detection plotted against the object's distance in meters for a diffuse object of reflectivity 1/10. The red line models optimal detection using entangled light, while the blue line models non-entangled light.

approximately 40 μRad, and the diameter of the receiver is 2 m. We will vary the size of the object with the type of scattering.

First, we will consider an object which scatters diffusely and has no specular component. In this case, we will assume a test object that is 1.2 m in diameter, and that we are sampling over 1s intervals. In Fig. 1, we plot the probability of detecting a test object of reflectivity 1/2 versus range; materials with similar reflectivity are ocean ice and matte aluminum [14]. We estimate that the probability of detecting our test object at 700m is a little over 80%. In Fig. 2, we consider a less reflective object and plot the probability of detection when the reflectivity is $1/10$. From Figs. 1 and 2, we estimate that at distances of 700 m and 400 m (respectively) the probability of detection is reduced by more than a factor of two when using non-entangled light versus entangled light. Note here, these figures only model the optimal detection method outlined in [3].

To demonstrate the extreme effect that the roughness of an object can have on the ability to detect it, we will model a test object which is smooth, so the scattering is specular. In this case we will assume the test object is smaller than the diffuse object, and has a diameter of 1 mm. Additionally, we will assume that we are sampling over 1 ms intervals. In Fig. 3, we plot the expected probability of detection when the reflectivity of the test object is 1/2. As shown in the plot, we estimate that the probability of detection at 8.2 km is approximately 80% using an optimal receiver. Even if we assume that the test object's reflectivity is much lower–since the scattering is specular–we estimate that we can still detect its presence with relativity high certainty at a range of kilometers. In Fig. 4, we plot the probability of detection for a specular object with a reflectivity of $1/10$; here, we estimate that we can detect the object with 80% efficiency at approximately 6.2 km using an optimal receiver. Comparing Figs. 1 and 2 to Figs. 3 and 4, the range for detecting a specular object significantly increases over that of an object that scatters diffusely. In addition, for the specular objects we modeled, entangled light could increase the range by up to a factor of 1.25 over non-entangled light (for a detection probability of 80%).

In general, most materials have surface imperfections bigger than 10 μm, unless they are made intentionally smooth, and therefore scatter an optical signal diffusely. As we have demonstrated, the diffuse scattering is severely range limiting. So although the first thought for this technology might be to probe an environment for the presence of an unknown object, we believe this technology might be well suited for a communication protocol, where the objects are made intentionally smooth, and therefore, have a greater range of detection.

V. A COMMUNICATION PROTOCOL

In each of the cases we considered, simply using a more powerful classical signal would greatly increase the range; so in using this technology, there must be some need to operate with low brightness. In Figs. 3 and 4, we considered moderate-to-low reflective surfaces where the scattering was mainly specular. We then wonder if a specular surface with low reflectivity could be used in an application for low power covert communication. In such a scenario Alice would transfer information to Bob, by obstructing photons sent from Bob in Alice's direction. Such a protocol would require very little power from Alice because she would not have to generate the signal that she is sending. Here, Alice sends information to Bob, but the information transfer would almost completely be powered by Bob: the presence of an object would allow Alice to send the classical bit 1, the absence would be a classical bit 0. In Fig. 4, we estimate that even if Alice used an object of reflectivity $1/10$, the optimal bound predicts Bob could receive Alice's message at a distance of 6.2 km with 80% certainty, and at a distance of 5.8 km using the more conservative suboptimal estimate. Our model further estimates that if Alice used an object with reflectivity 1/100, Bob could still receive her message with 80% efficiency at a range of 3.7 km (not plotted).

Objects like weighted beam splitters and optical windows, offer the ability to transmit a certain percentage of light

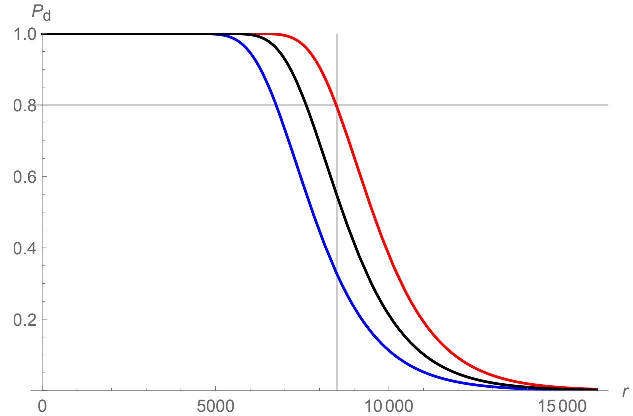

Fig. 3. Probability of detection plotted against the object's distance in meters for a specular object of reflectivity 1/2. The red line models detection using an optimal receiver and the black line models a suboptimal receiver, both using entangled light. The blue line models non-entangled light.

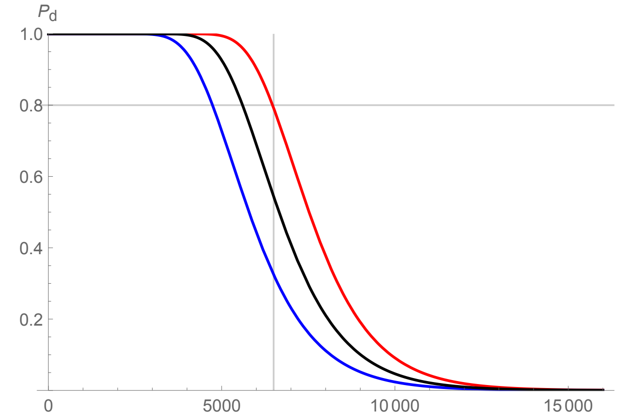

Fig. 4. Probability of detection plotted against the object's distance in meters for a specular object of reflectivity 1/10. The red line models detection using an optimal receiver and the black line models a suboptimal receiver, both using entangled light. The blue line models non-entangled light.

while specularly reflecting the rest; we envision Alice using an object like that. Since most objects reflect diffusely, the system couldn't detect them at long distances. Therefore, any object that our system detected at these distances should be Alice's optical element. In addition, at this level, the contributions from the signal are indistinguishable from environmental noise. Combining that with the fact that Alice's object is small, and almost indistinguishable from a clear optical window, we opine that Alice could communicate with Bob in a covert nature.

VI. Conclusion

The technology for generating entangled states is far more developed in the optical regime than for longer wavelengths, but using such a short wavelength undoubtedly limits the range at which one can detect the presence of an object. Both the atmospheric attenuation and the relative roughness of most objects are range limiting factors in the optical regime. We believe one of the most conspicuous challenges in significantly extending the detection range for a diffuse object is to develop the corresponding technology for much longer wavelengths. If such technology existed, we expect that the range would dramatically increase for objects that appear "rough" to an optical signal. However, this technology is in its infancy, and is not capable of producing the quantity of entangled pairs that one would need for an experiment that we modeled. In order to operate in the microwave wavelengths, for example, significant developments in generating and detecting microwave entangled states are needed.

We acknowledge that there are many engineering challenges to implement our proposed communication protocol, especially the challenge of building a receiver that achieves the optimal detection probability. Extending the range, accuracy, and bitrate requires the fast and efficient generation of entangled photons in distinguishable optical modes, and highly efficient single photon detectors with low dark counts. Increasing the pump power on the PPKTP crystal will generate more pairs, but at higher powers one has to consider the effect of multi-pair generation. Therefore simply increasing the pump power may not be a viable strategy for creating a better signal. In addition to the practical challenges, there are still many aspects of this protocol to consider. Some variables to consider augmenting are the reflectivity of Alice's optical element and the size of Bob's transmitter and receiver.

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