Changes from 2011 NRL Plasma Formulary to the
2013 NRL Plasma Formulary

▷ p. 37

electrical conductivities changed from

$$\sigma_{\parallel} = 1.96\sigma_{\perp}; \quad \sigma_{\perp} = ne^2\tau_e/m_e$$

to (from Balescu, ref 21b)

$$\sigma_{\parallel} = 1.96\sigma_0; \quad \sigma_{\perp} = \sigma_0(\omega_{ce}\tau_e)^{-2}; \quad \sigma_0 = ne^2\tau_e/m_e$$

NOTE: Unfortunately the above change has resulted in a misrepresentation of the original Braginskii results as noted by several plasma physicists. This is due to the usage and meaning of the word 'conductivity.' The above change is based on the interpretation of conductivity as an electrical conductivity relating the current to the electric field ($\mathbf{J} = \sigma \cdot \mathbf{E}$). However, Braginskii's usage of conductivity refers to the inverse of the frictional resistivity in velocity equation on p. 36. A better description of these conductivities is the following:

$$\sigma_{\parallel}^{e,r} = 1.96\sigma_{\perp}^{r}; \quad \sigma_{\perp}^{r} = ne^2\tau_e/m_e; \quad \sigma_{\perp}^{e} = \sigma_{\perp}^{r}(\omega_{ce}\tau_e)^{-2}$$

where the superscripts e and r refer to electrical and resistive, respectively.

▷ p. 44

fusion reaction (5c) changed from

... He^5 (2.4 MeV) + p (11.9 Mev)
to
... He^5 (1.89 MeV) + p (9.46 Mev)

▷ p. 35

the factor 35 should be 43 in the expression for $\lambda_{ii'}$ (see following page). this change *has not been made* in the 2013 NRL plasma formulary.

Coulomb logarithm for counter-streaming ions in the presence of warm electrons

Scott C. Hsu
Physics Division, Los Alamos National Laboratory, Los Alamos, NM 87545

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We re-derive the Coulomb logarithm for counter-streaming ions in the presence of warm electrons and obtain a different result than the one given in the NRL Plasma Formulary (2013 edition).

I. RE-DERIVATION OF THE COUNTER-STREAMING ION-ION COULOMB LOGARITHM

The Coulomb logarithm for counter-streaming ions with relative velocity $v_D = \beta_D c$ in the presence of warm electrons ($kT_i/m_i, kT_{i'}/m_{i'} < v_D^2 < kT_e/m_e$) is given in the NRL Plasma Formulary (2013 edition, top of p. 35) as

$$\lambda_{ii'} = \lambda_{i'i} = 35 - \ln\left[\frac{ZZ'(\mu+\mu')}{\mu\mu'\beta_D^2}\left(\frac{n_e}{T_e}\right)^{1/2}\right], \quad (1)$$

where T_e is in eV and cgs units are used throughout, consistent with the convention in the NRL Plasma Formulary. Unprimed and primed variables correspond to test and field particles, respectively.

In some cases that are clearly weakly coupled, $\lambda_{ii'}$ falls below unity as calculated using Eq. (1). For example, for counter-streaming Al-Al collisions with $\mu = \mu' = 27$, $Z = Z' = 2.0$, $v_D = 20$ km/s, $T_e = 2.2$ eV, and $n_e = 6.5 \times 10^{14}$ cm^{-3}, $\lambda_{ii} = 0.325$. This prompted us to check Eq. (1) for accuracy.

The definition of the Coulomb logarithm is

$$\lambda = \ln\Lambda = \ln\left(\frac{r_{\max}}{r_{\min}}\right), \quad (2)$$

where in this case

$$r_{\max} = \lambda_{De} = \left(\frac{kT_e}{4\pi n_e e^2}\right)^{1/2} = 7.43 \times 10^2 \left(\frac{T_e}{n_e}\right)^{1/2} \text{ [cm]} \quad (3)$$

and

$$r_{\min} = b = \frac{ZZ'e^2}{m_{ii'}v_D^2} \text{ [cm]}, \quad (4)$$

where b the distance of closest approach between two counter-streaming ions with reduced mass $m_{ii'} = m_i m_{i'}/(m_i + m_{i'})$ and relative speed v_D. Here, we assume that b is greater than the de Broglie wavelength $\hbar/m_{ii'}v_D$.

We re-write b by pulling out numerical constants:

$$b = \frac{e^2}{m_p c^2}\frac{(\mu+\mu')ZZ'}{\mu\mu'(v_D/c)^2} = \frac{(4.8032\times10^{-10})^2}{(1.6726\times10^{-24})(2.9979\times10^{10})^2}\frac{(\mu+\mu')ZZ'}{\mu\mu'\beta_D^2} = 1.5347\times10^{-16}\frac{(\mu+\mu')ZZ'}{\mu\mu'\beta_D^2}. \quad (5)$$

Plugging Eqs. (3) and (5) into Eq. (2), we obtain

$$\lambda_{ii'} = \ln\Lambda = \ln\left[\frac{743(T_e/n_e)^{1/2}}{1.5347\times10^{-16}\frac{(\mu+\mu')ZZ'}{\mu\mu'\beta_D^2}}\right] = 43 - \ln\left[\frac{ZZ'(\mu+\mu')}{\mu\mu'\beta_D^2}\left(\frac{n_e}{T_e}\right)^{1/2}\right]. \quad (6)$$

The discrepancy between the numerical factors 43 [in Eq. (6)] and 35 [in Eq. (1)] is exactly accounted for if the constants $k = 1.6022 \times 10^{-12}$ erg/eV and $e^2 = (4.8032\times10^{-10})^2$ statcoul^2 in Eq. (3) are ignored (thanks to G. Swadling of Imperial College for pointing this out), which would change the numerical factor of 743 to 0.282 in Eqs. (3) and (6). This seems like a plausible mistake to make in the original derivation of Eq. (1).

Using the Al-Al parameters given earlier, we get $b = 1.02 \times 10^{-8}$ cm, $\lambda_{De} = 4.32 \times 10^{-5}$ cm, and $\lambda_{ii} = \ln\Lambda = 8.1$, a more reasonable result for our weakly coupled plasma.

In summary, we propose that Eq. (6) is the more correct expression for the Coulomb logarithm for counter-streaming ions in the presence of warm electrons.